Dynamics of life expectancy and life span equality

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As people live longer, ages at death are becoming more similar. This dual advance over the last two centuries, a central aim of public health policies, is a major achievement of modern civilization. Some recent exceptions to the joint rise of life expectancy and life span equality, however, make it difficult to determine the underlying causes of this relationship. Here, we develop a unifying framework to study life expectancy and life span equality over time, relying on concepts about the pace and shape of aging. We study the dynamic relationship between life expectancy and life span equality with reliable data from the Human Mortality Database for 49 countries and regions with emphasis on the long time series from Sweden. Our results demonstrate that both changes in life expectancy and life span equality are weighted totals of rates of progress in reducing mortality. This finding holds for three different measures of the variability of life spans. The weights evolve over time and indicate the ages at which reductions in mortality increase life expectancy and life span equality: the more progress at the youngest ages, the tighter the relationship. The link between life expectancy and life span equality is especially strong when life expectancy is less than 70 y. In recent decades, life expectancy and life span equality have occasionally moved in opposite directions due to larger improvements in mortality at older ages or a slowdown in declines in midlife mortality. Saving lives at ages below life expectancy is the key to increasing both life expectancy and life span equality.

aging | demography | life span variation | mortality | pace and shape

The rise in human life expectancy over the past two centuries is a remarkable accomplishment of modern civilization (1, 2). This progress was achieved during the demographic transition of societies from regimes of high mortality and fertility to regimes of low mortality and fertility (3, 4). At present, among the world's nations, Japanese women have the highest life expectancy at birth, above 87 y. In 1840, the record was held by Swedish women, with an average life span of 46 y (5). This advance has been accompanied by an increase in life span equality: In low mortality populations today, most individuals survive to similar ages (6–11).

Life span equality matters because it captures a fundamental type of inequality: variation in length of life. This variation is not revealed by life expectancy and other measures of average mortality levels (12). Two populations that share the same level of life expectancy could experience substantial differences in the timing of death, e.g., deaths could be more evenly spread over age in one population than another. Although life expectancy is monitored in every country in the world, few countries have begun to monitor and acknowledge the importance of disparities in age at death.

For values of life expectancy at birth from under 20 to above 85 y, life span equality rises linearly (Fig. 1). This relationship between life expectancy and life span equality has been found to hold in a life span continuum over millions of years of primate evolution, in many countries and among subgroups in a population (6–11, 13–15). The dual advance, however,

might be coincidental rather than causal. Even though both life expectancy and life span equality are computed from the same information, namely age-specific death rates, doubt about a common causal link is sown by messier and sometimes negative relationships between them in various datasets and using alternative indicators of life span equality (16). The United States, for example, has relatively low equality in life spans in comparison with other countries that have similar levels of life expectancy (17). Scotland reached similar levels of life expectancy with 10% higher life span inequality than England and Wales since 1980 (18). Finnish females from lower educational levels experienced increases in life expectancy, while life span equality decreased at age 30 since the 1970s (12). In Denmark, life span equality decreased among the lowest income subgroup over the period of 1986 to 2014 despite the increase in life expectancy (19). In some countries in Eastern Europe and Latin America, life expectancy and life span equality moved independently over periods of slow improvements in life expectancy (20–22). Indeed, in many countries and subgroups within a country in recent decades, life span equality declined, although the average life span rose or vice versa (as indicated by the points in the second and fourth quadrants of Fig. 2 *A* and *B*). In addition, causes of death that
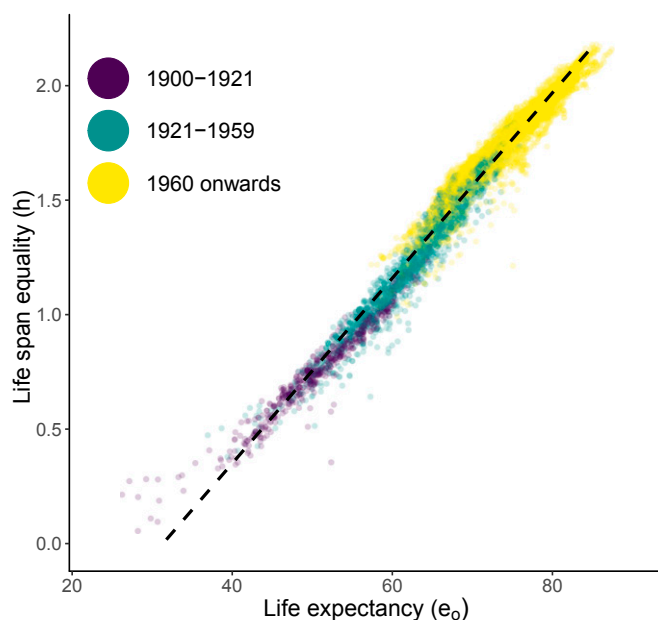
Fig. 1. Association between life expectancy at birth e_o and life span equality h.

contributed to increasing life expectancy somewhat differ from those that increased equality in life spans in developed countries after 1970 (23, 24). Nonetheless, despite these exceptions and discrepancies, life expectancy and life span equality generally move in the same direction (11).

In this article, we develop a mathematical framework to explore how life expectancy at birth and life span equality relate to each other and evolve over time. We rely on two dimensions of aging: the average length of life (pace) and the relative variation in length of life (shape) (25). The former refers to how fast aging occurs, while the latter describes how sharply populations age. The shape of mortality pertains to the distribution of life spans. Statisticians and demographers, based on both theoretical and practical considerations, have developed different indicators to summarize the distribution of life spans (26, 27). Here, we measure average length of life by life expectancy, and we analyze the distribution of life spans with three different indicators of life span equality. These indicators are variants of 1) the life table entropy, 2) the Gini coefficient, and 3) the coefficient of variation of the age-at-death distribution (28, 29). Other indicators of

absolute dispersion in life spans exist, such as the variance of the age-at-death distribution, its SD, or life years lost due to death (30, 31). However, these are pace indicators measured in units of time and do not capture the dimensionless shape of aging (26).

We focus on how age-specific mortality improvements change life span equality and life expectancy at birth. We analyze changes over time in these two longevity measures for Swedish females since the 18th century, and 48 additional populations from the Human Mortality Database with reliable data, in many cases since the beginning of the 20th century, for females and males separately (5). Mortality risks implied by a period life table generally differ from the risks individuals will experience over their lifetimes. Nonetheless, life table information on life expectancy and life span equality may provide information individuals use to make life course decisions, and information policymakers use to assess population health and well-being (32–34).

Trends in Life Expectancy and Life Span Equality

Life expectancy at birth for both men and women increased throughout the 20th century (5, 35). Paralleling the rise of life expectancy, all countries included in our study became more equal in life spans (Fig. 1). This is a significant advance in giving people more equitable opportunities. Furthermore, the rise in life span equality has influenced the decisions individuals make over their life course, such as when to have children, study, work, or retire, and how much to save for retirement, because such decisions are based not only on expected lifetime but also on uncertainty about age at death (14). Analysis of the relationship between life expectancy at birth e_o and life span equality, as measured by h, a log-transformation of life table entropy $\overline{H}$ (*Materials and Methods* and Box 1), indicates a strong correlation (Pearson coefficient of 0.985 for the data in Fig. 1). We also analyzed the relationship between average life span and two other measures of life span equality based on the Gini coefficient and the coefficient of variation, and found similarly high correlations, 0.981 and 0.975, respectively (*SI Appendix*, Fig. S1). Although life expectancy and life span equality have been positively correlated, it is apparent that the relationship is less strong and often reversed in recent decades, resulting in negative correlations in some countries in yearly and 10-y changes (Fig. 2).

How Strong Is the Relationship Between Life Expectancy and Life Span Equality over Time?
To study how strongly life expectancy and life span equality are related over time and whether they respond in the same direction to age-specific mortality changes, we complement demographic analysis with time series analysis (see *SI Appendix*, section A for details). This framework is
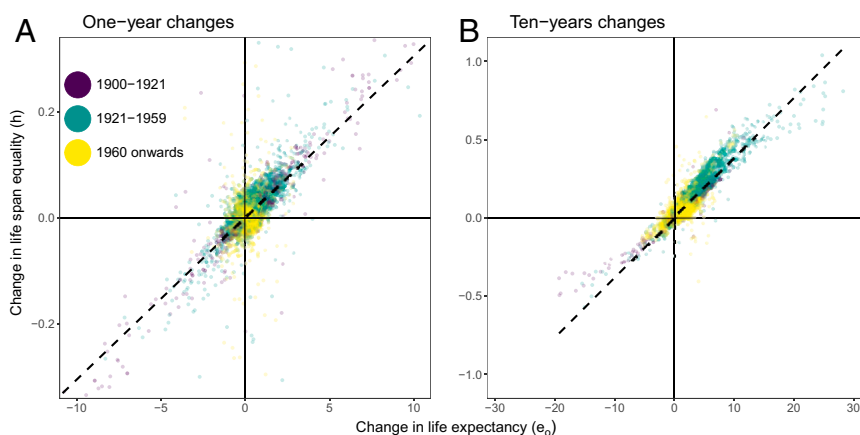
Fig. 2. (A) Association between changes in life expectancy at birth e_o and life span equality h. (B) Association between changes over 10-y rolling periods.

designed to integrate the stochastic properties of dynamics over time (8, 9). Focusing on changes over time improves our analysis by avoiding misleading inferences from correlations, such as confounding due to unobserved or unmeasured variables (36). Econometric time series theory indicates that life expectancy and life span equality have a long-run relationship if there exists a single process that drives both indicators toward a long-term equilibrium, even if temporary departures from it occur (as observed more often in recent decades). If this equilibrium exists, changes over time in life span equality are proportional to changes in life expectancy in the long term. In other words, while life expectancy and life span equality increase over time, a linear combination of both leads to a residual time series consistent with stationarity (i.e., with stable mean and variance), referred to as cointegration in time series analysis (*SI Appendix*, section A.2).

The results reveal that, in most populations, life expectancy and life span equality are linked by a long-run relationship for both sexes (*SI Appendix*, Fig. S2). In 91% of the populations we investigated (males and females from 45 countries and regions by sex), this relationship holds under the same model specifications (*SI Appendix*, section A.2); similar results are exhibited for all three indicators of life span equality (*SI Appendix*, Fig. S2). At the 5% significance level, negative results are expected for 5% of the cases due to random variations. We got negative results in 9% cases. So, the importance of negative results in specific populations should not be overly emphasized (*SI Appendix*, section A. 3). These results hold for countries that have experienced substantially different mortality patterns, including women in Japan; men in the United States with life expectancy of about 77 y and relatively high life span inequality (17); and men in Russia and

Ukraine with the lowest levels in life expectancy in this study (about 65 and 66 y in 2013, respectively) and high inequality (21). Importantly, for every population in our study, females' lives tend to be longer and more equal compared to males' lives in a given year, consistent with previous research (11, 37). This underscores the advantage of females over males not only in average life span but also in lower uncertainty about age at death.

Age-Specific Dynamics of Mortality. The field of demography has long been known within the social sciences for its innovations in decomposition analysis (38). Decomposition analysis is based on the principle of separating demographic measures, e.g., life expectancy or life span equality, into components that contribute to their dynamics, such as age-specific mortality. Several methods to analyze change in life expectancy over time have been developed. Pollard (39), Arriaga (40), and Andreev et al. (41), among others, focused on discrete differences in life expectancy, while other authors considered continuous change (42–46). Some of these methods have been extensively used in population health studies to disentangle age- and cause-specific effects because they are easy to implement (40, 47, 48). Here, we relate changes in both life expectancy and life span equality to the average pace of improvement of mortality and the average number of years lost at death (31). We are able to describe specific properties of both indicators.

Changes in life expectancy and in life span equality over time are weighted averages of rates of progress in reducing age-specific mortality, $\rho(x)$, albeit with different weights (*Materials and Methods*). These weights—$w(x)$ for life expectancy at birth and the product $w(x)W_h(x)$ for life span equality—evolve over time and vary by age. They indicate the potential gain (loss) in
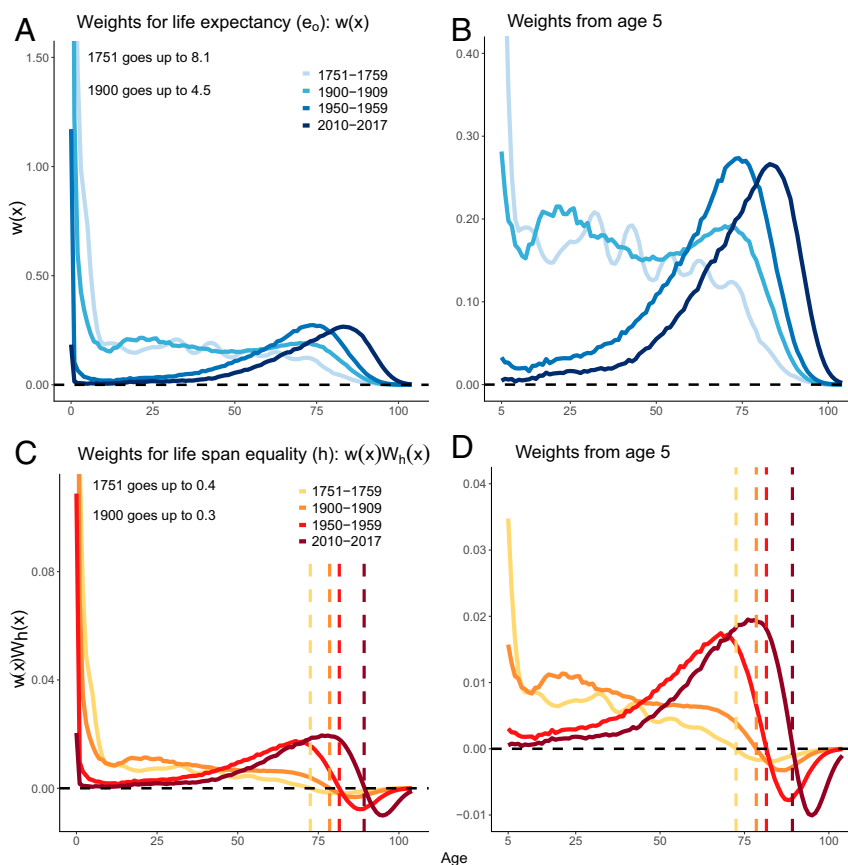

Fig. 3. Weights for the changes in life expectancy $w(x)$ (*A* and *B*) and life span equality $w(x)W_h(x)$ (*C* and *D*). Each line refers to a given period and represents how life expectancy and life span equality react to age-specific mortality improvements for Swedish women in selected periods.
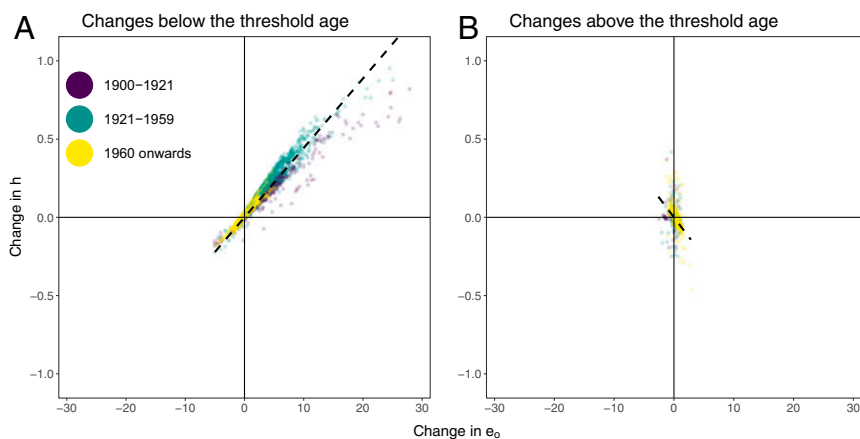
Fig. 4. (*A*) Association between 10-y changes in life expectancy at birth *e*$_o$ and life span equality *h* because of mortality changes below the threshold age. (*B*) Association between 10-y changes in *e*$_o$ and *h* because of mortality changes above the threshold age. The dotted lines show the directions of the relationship below and above the threshold age.

life expectancy and life span equality if lives are saved at a specific age and in a given period. Fig. 3 *A* and *B* shows the weights for life expectancy at birth and from age 5 for Swedish women. From the 18th to the first part of 20th century, the largest potential increases in life expectancy were concentrated in infancy. The effect on life expectancy improvements due to saving lives in midlife was higher than at older ages. This changed dramatically after 1950, when the effect of infant mortality decreased significantly. By 2010, the effect of reducing mortality by 1% at birth was the same as reducing mortality by 1% at age 71. In the 21st century, saving lives between ages 5 and 40 y had a negligible effect on life expectancy, as opposed to the relatively high impact of these ages before 1900. A shift over time toward the importance of older ages is clear. This ongoing wave toward older ages is in line with recent evidence documenting an advancing front of old-age survival that has driven recent increases in average life span (49). Indeed, the postponement of old-age mortality is an ongoing process that started more than 50 y ago (50, 51). Fig. 3 *A* and *B* shows that whenever mortality improvements occur life expectancy increases. The size of the increase depends on the ages at which lives are saved. These improvements $\rho(x)$ and the weights $w(x)$ are the drivers of life expectancy at birth over time (31). Fig. 3 *C* and *D* shows the weights $w(x)W_h(x)$ for life span equality *h*. As in *A* and *B*, each value represents the effect on life span equality of reducing mortality at a given age. Saving lives at very young ages had the largest effect on increasing equality of life spans throughout the 18th, 19th, and first half of the 20th centuries. In contemporary Sweden, the impact of reducing mortality at birth on life span equality is the same as saving lives at all ages between 76 and 80 y.

As with life expectancy, there is an ongoing shift toward older ages, but with an important difference. At older ages, there is a threshold age above which saving lives decreases life span equality (Box 1). This age is depicted with the dashed lines colored according to the respective period: An increase of this age over time clearly appears in the graphs. The threshold age gives valuable information for understanding of the relationship between life expectancy at birth and life span equality: To the extent that improvements at ages below the threshold age outpace those above it, life expectancy will move in the same direction as life span equality (52). The shift from positive to negative effects has previously been explored using other indicators (53, 54). The three life span equality indicators that we analyze behave similarly (*SI Appendix*, Fig. S3); their sensitivity to changes in age-specific mortality resembles that of other indices of life span variation (27).

Fig. 4*A* shows the contributions, in years, of mortality fluctuations below the threshold age (early component), and Fig. 4*B*

shows contributions above the threshold age (late component) to changes in life expectancy and life span equality in 10-y rolling periods for all countries included in our study. The points in the first and third quadrants in Fig. 4*A* and the second and fourth quadrants in Fig. 4*B* reflect a mix of reductions in death rates at some ages below and above the threshold and increases at other ages. Because the weights for specific ages differ for life expectancy and life span equality, the aggregate effect of such a mix of mortality changes can be positive (negative) for life expectancy and negative (positive) for life span equality. The sum of the early and late components gives the total change in each indicator (Fig. 2*A*). We report similar results for the two other indicators of life span equality in *SI Appendix*, Fig. S4. There is a strong positive association between changes in life expectancy and life span equality below the threshold age, while the relationship is negative above that threshold. Since the two effects oppose each other, as shown by the regression lines, the relationship is driven by the component that makes the larger contribution. Reductions in death rates below the threshold age were significantly larger than reductions above it before 1960, resulting in a strong positive association between life expectancy and life span equality. Since 1960, mortality reductions above the threshold age have become more comparable in magnitude to the early-life component, with more increases in life expectancy coinciding with decreases in life span equality. Until now, the absolute change in both indicators per decade is mainly driven by mortality changes below the threshold age (83.7% and 82.0% on average per decade for life span equality and life expectancy, respectively [Fig. 4 and Box 1, Fig. 1 *B* and *C*]).

As life expectancy increases, the threshold age also increases (Box 1 and *SI Appendix*, Fig. S5). There is then more scope to save early lives below the threshold age and maintain the positive relationship between life expectancy and life span equality. This is an essential characteristic of the long-run equilibrium. Progress, however, after the threshold age has been increasing. For example, in Sweden the most common age at death at older ages was stagnant up until the 1950s when it started rising with life expectancy (Box 1, Fig. 1*A*), and contributions to changes in life expectancy and life span equality increased above the threshold age (Box 1, Fig. 1*C*). These results underscore the effect of mortality improvements at advanced ages (i.e., above the threshold age) in recent years and shed light on recent interruptions in the relationship between changes in life expectancy and life span equality. This process follows a redistribution of mortality over age and causes of death (23, 55, 56). In the past, deaths were concentrated at young and working ages, mainly due to infectious diseases and to

Box 1. The Threshold Age and the Life Expectancy at Birth

Life span equality measured by h refers to an indicator closely related to the life table entropy, which was first developed by Leser (29) and further explored by Demetrius (62), Keyfitz (42), and Keyfitz and Golini (63). The life table entropy is a dimensionless indicator of the relative variation in the length of life compared to life expectancy at birth, and can be expressed as follows:

$$\overline{H} = -\frac{\int_0^\infty \ell(x)\ln\ell(x)dx}{\int_0^\infty \ell(x)dx} = \frac{e^\dagger}{e_o}.$$

Function $\ell(x)$ denotes the probability of surviving from birth to age x, whereas $e^\dagger$ refers to life disparity—the average remaining life expectancy at ages of death (31, 45, 46)—and e_o is the life expectancy at birth.

Life span equality measured by $h = -\ln\overline{H}$ has previously been used as an indicator of life span equality (11). If mortality improvements over time occur at all ages, there exists a unique threshold age that separates positive from negative contributions to $\overline{H}$ as a result of those improvements (52). Because h is a logarithmic transformation of $\overline{H}$, it has the same threshold age, which we denote by a^h (vertical dashed lines in Fig. 3). This threshold is reached when

$$H(a^h) + \overline{H}(a^h) = 1 + \overline{H},$$

where $H(a^h)$ is the cumulative hazard to age a^h and $\overline{H}(a^h)$ is the life table entropy conditional on surviving to age a^h (52).

Box 1, Fig. 1 shows the evolution of life expectancy at birth e_o, the threshold age a^h, and the most common age at death after infancy, M, for Swedish females since 1900 (A). The figure highlights how the three measures move together. The threshold age in A is the age that separates "early" from "late" deaths in terms of the effect on life span equality. Averting deaths before a^h increases equality, while averting deaths after this age has the opposite effect. It is a population-specific measure that depends on the observed mortality pattern. The threshold age and the life expectancy at birth move in the same direction, either increasing or decreasing together; note that they are very close in recent decades. The modal age at death M was fairly constant before 1950 and rose in tandem with e_o and a^h thereafter. More than 40% of deaths occur below e_o and a^h, whereas more than 60% of deaths occur below M (B). C and D show that mortality improvements below e_o and a^h were responsible for around 80% of gains in life expectancy at birth and life span equality in the beginning of the 20th century, while they are responsible for around 60% in contemporary Sweden.
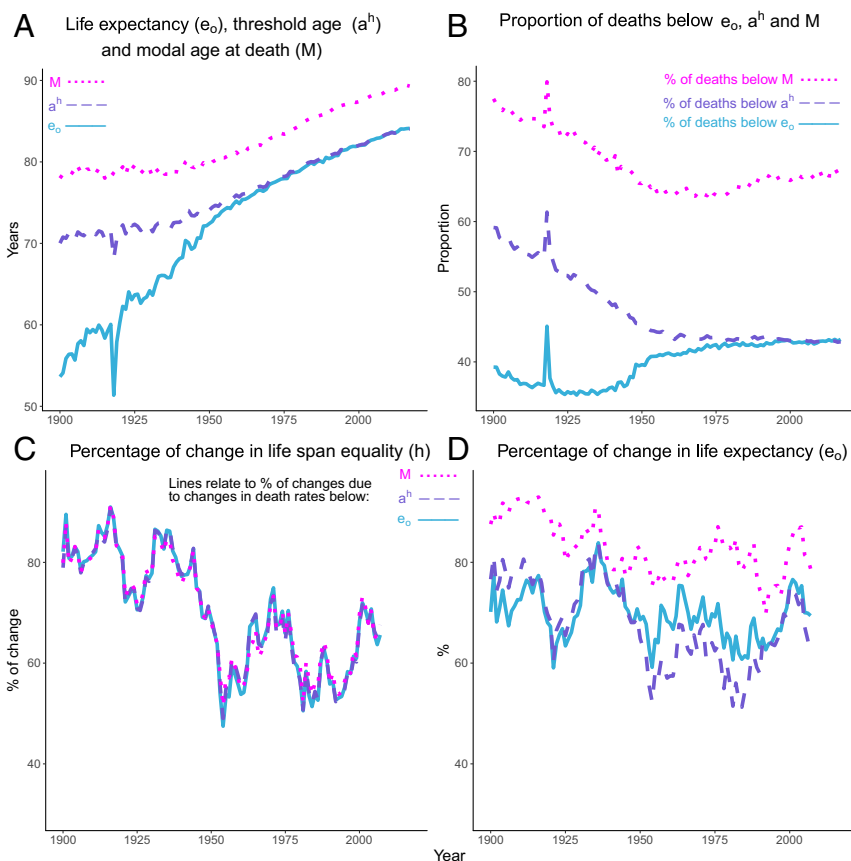
Box 1, Fig. 1. (*A*) Life expectancy at birth e_o, threshold age a^h, and modal age at death M. (*B*) Proportion of deaths below life expectancy at birth e_o, threshold age a^h, and modal age at death M. (*C*) Percentage of changes in life span equality due to changes in death rates below life expectancy at birth e_o, threshold age a^h, and modal age at death M. (*D*) Percentage of changes in life expectancy at birth due to changes in death rates below life expectancy at birth e_o, threshold age a^h, and modal age at death M.

some extent wars and famines that resulted in high inequality of life spans (57). In recent decades, because of major improvements in health services and medical treatment, living standards, sanitation, and various social determinants of health (58–61), lifesaving is concentrated at older ages, sometimes above the threshold age.

Outlook. The dynamics of life expectancy and of life span equality are driven by changes in age-specific death rates. The impact of the change at some age differs somewhat for the two measures. At younger ages, the impacts are similar. After a threshold age late in life, a reduction in age-specific death rates increases life expectancy but decreases life span equality. Because of progress in recent decades in reducing death rates above the threshold age, rises in life expectancy more often coincide with declines in life span equality. For the populations we analyzed, in the period 1900 to 1950 less than 16% of the annual changes in average life span coincided with opposite changes in life span equality. In the 1960s, this discrepancy rose to 47%; and thereafter the average has been around 32%. These trends were driven by Eastern and Central European countries and by Nordic countries, which experienced divergent patterns in mortality at different ages (21, 24). Since 1960, life span inequality was high and fluctuated strongly in Central and Eastern Europe. A recent study shows that in the decades 1960 to 1980, life expectancy and life span equality changed in opposite directions in half the years and populations analyzed, largely because trends in age-specific death rates were positive at some ages and negative at other ages (21). This is consistent with our findings. Previous evidence suggests that alcohol-related and cardiovascular diseases might have been important in driving the observed trends in Central and Eastern Europe (21, 64–66). Danish males experienced deterioration caused by smoking-related and cardiovascular mortality between ages 35 and 80, while negative trends in Norway and Sweden were mostly caused by an increase in cardiovascular mortality (24).

Are there paths other than the joint, linear rise in Fig. 1 that might have been followed if social conditions and public policies had been different? This is an intriguing question that can be examined in our framework. Fig. 5 shows the relationship between life expectancy and life span equality for Swedish women from 1751 to 2017 under three different scenarios. Blue points refer to observed life expectancy from values below 20 y in 1773 to 84.1 y in 2017. The process of increasing life expectancy with greater equality in individual life spans has been referred to as the compression of mortality or the rectangularization of survivorship, and has been studied from various perspectives over the last couple of decades (7–11, 21, 67). Understanding the dynamics of the compression of mortality is important for forecasting heterogeneity in future age patterns of population health as well as for assessments of the timing of individual mortality (12).

Consider the difference of life expectancy and life span equality between two consecutive years. The regression line in Fig. 5 indicates that the average change in life expectancy is about 25.4 times the life span equality change, a value close to the 27 reported elsewhere (11). Here, we demonstrate that each of these first differences, as an approximation to the time derivative (*Materials and Methods*), is a weighted total of mortality improvements in a given year (Fig. 3). Our main motivation lies on the remarkably tight relationship between life expectancy and life span equality through time illustrated by the regression line (slope, 0.04; $P < 0.001$). For example, in 1773 Sweden underwent the last major famine that caused starvation across the country (68). Approximately 50% of excess deaths were due to dysentery, and most deaths (20%) were concentrated in infancy (57). Even under periods of such mortality stress, observed life expectancy and life span equality fall on the linear trend that holds in more favorable years. Is this tight connection coincidental or a result of fundamental social and physiological forces? We have shown that the
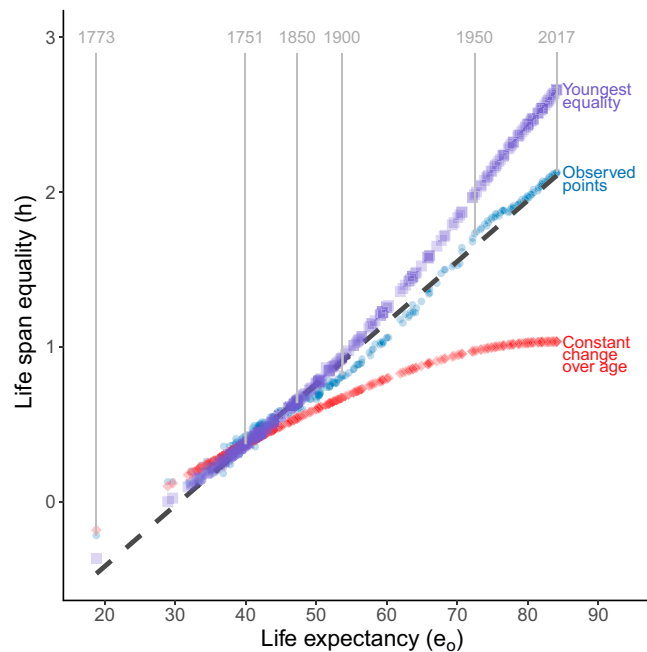

Fig. 5. Life expectancy at birth e_o and life span equality h for three different scenarios: 1) observed points: Swedish females, 1751 to 2017; 2) youngest equality: life span equality derived by matching observed life expectancy levels by reducing the youngest age; and 3) constant change over age: death rates in each year at all ages are reduced at the rate ρ to achieve the observed change in life expectancy at birth.

connection is largely due to change in death rates at younger ages. Can more be said?

The observed path (blue points, Fig. 5) is a combination of age-specific mortality improvements and the weights shown in Fig. 3. Improvements in mortality are uneven across ages (31). Hence, we explored an alternative scenario in which the same rate of mortality reduction (or increase) occurred at all ages, the "constant scenario," with the rate chosen to be consistent with observed levels of life expectancy over time. The red rhombuses in Fig. 5 illustrate the resulting trajectory for Sweden. When the average life span rises above 40 y, levels of life span equality start to diverge and become lower than the observed ones. The relationship between life expectancy becomes nonlinear and levels off at around a life expectancy at birth of 70 y.

Another hypothetical scenario is represented by the purple squares labeled "youngest equality." This curve refers to the case when all progress in reducing death rates is concentrated at the youngest ages. For example, to get the 1752 life expectancy level from 1751, only deaths at age zero are reduced. Then when deaths at birth are zero, deaths are reduced at age 1, then age 2, and so on, to match the observed life expectancy in the following years. That is, all lifesaving is concentrated at the youngest age(s) at which deaths still occur. Results yield a steeper slope (0.051; $P < 0.001$), which translates into larger equality in individual life spans at levels of life expectancy after age 50.

Consider now another scenario, the "potential scenario." From the level of life expectancy in 1950 to contemporary Sweden, age-specific rates of improvement are chosen such that 1) life expectancy increases continuously match the observed levels every decade, and 2) life span equality increases optimally. That is, when life expectancy increases, progress is concentrated at the ages when change in death rates most increases life span equality. Also consider the "constant scenario" in which the life expectancy improvement every decade was achieved by reducing mortality at the same rate for every age. Table 1 shows life span equality under

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Table 1. Life expectancy at birth e_o and life span equality h for three different scenarios

Year	e_o	Life span equality h by scenario			(Observed − Constant)/ (Potential − Constant), %
		Observed	Potential	Constant	
1960	74.88	1,84	1,90	1,76	57
1970	77.21	1,87	1,99	1,86	8
1980	78.86	1,93	1,98	1,88	50
1990	80.39	1,98	2,03	1,94	44
2000	82.01	2,05	2,09	1,99	60
2010	83.47	2,11	2,15	2,05	60
2017	84.12	2,13	2,16	2,11	40

The three different scenarios are as follows: 1) observed points: Swedish females, 1960 to 2017; 2) potential equality: life span equality derived by matching observed life expectancy levels by reducing death rates that increase life span equality the most; and 3) constant change in mortality improvements $\rho(x)$ over age matching observed life expectancy levels every decade.

these scenarios for Swedish females from 1960 as well as the actual observed trajectory of life span equality. The potential scenario leads to the highest attained life span equality, while the constant scenario shows the lowest equality in life spans. Interestingly, what was observed in Sweden is close to 50% on average of the difference between the potential and constant scenarios. Hence, the observed trajectory might be called the "semioptimal scenario." These alternative scenarios show that the narrow passageway that describes the relationship between life expectancy at birth and life span equality is not a coincidence. The transition from low levels of average life span and high variation in length of life to longer and more equal life spans is a result of saving lives at ages that matter—but semioptimally. The tight link between life expectancy and life span equality has been shaped by improvements in mortality at the most important ages for life expectancy and for life span equality: early ages in the 18th century and adult ages today.

In recent years, more instances of a temporary reversal of the relationship between life expectancy and life span equality have been observed in several countries and subgroups of populations (12, 20–22). Often these cases were due to midlife mortality deterioration or to major improvements in old-age mortality above the threshold age. In Sweden, death rates among octogenarians and nonagenarians have fallen since 1950 (69). For other developed countries, the pattern has been similar (70). If improvements at advanced ages continue and if they outpace those made at younger ages, the pattern of the relationship between life expectancy and life span equality could reverse in the future. It is, however, unlikely that rates of improvement above the threshold age will outpace progress at younger ages in the long term. Furthermore, as life expectancy increases, the threshold age will increase.

Across primate species, there is a rough association of life expectancy and life span equality. Several instances, however, where a relationship between the pace and shape of aging is not found have been documented in other species. Across the tree of life, 46 diverse species did not show a strong correlation between life expectancy and life span equality (71), and among plants a nonlinear, but weak, positive association has been reported (72). These findings compare different species, whereas our results are for a single species in a changing environment. Two studies, one of the nematode worm *Caenorhabditis elegans* and the other of *Drosophila melanogaster*, of individuals held under different conditions, found that life span equality appeared to be independent of life expectancy (73, 74).

For humans, a sharp worsening of conditions tends to lead to substantial increases in infant and child mortality (57), and in some cases mortality at young adult ages, e.g., as experienced in

the former Soviet Union after the end of the anti-alcohol campaign and the dissolution of the USSR (21), lowering both life expectancy and life span equality. On the other hand, improvements in standards of living, nutrition, education, public health, and other environmental conditions tend, at least when life expectancy is less than 70, to predominantly affect life expectancy—and life span equality—through reductions in death rates at young ages (2).

A key question is whether changes in environmental conditions have their biggest effects on mortality in infancy and childhood because of human agency or because of human physiology. Do societies act to focus mortality improvements at the ages that matter the most, or is human mortality for physiological reasons most sensitive at younger ages to environmental changes? Study of the impact of environmental change on life expectancy and life span equality in nonhuman primate species, being undertaken by Fernando Colchero, Susan Alberts, and colleagues, could shed light on the role of agency versus physiology. More generally, our findings—coupled with the mathematical relationships we derived to analyze how changes in age-specific death rates affect life expectancy and life span equality—suggest that a link may be found for species in which environmental change affects life expectancy largely because of changes in death rates at young ages.

Materials and Methods

Data. We used death rates by age and sex from the Human Mortality Database (5) for 49 countries and regions by single age and year, with data available from the beginning of the 20th century for some of the countries and regions and later in the 20th century for others and with data up to the most recent year available (see *SI Appendix*, Table S1 for detailed information). We constructed life tables following standard demographic procedures (7,717 life tables) (75). For each population, we investigated life expectancy at birth and life span equality by sex. The analysis is restricted to countries with data available for consecutive years (without gaps in the information over time) in order to study age-specific mortality patterns on a yearly basis. We decided not to analyze dispersion at death conditional on survival to any older age because of major improvements made in early ages during the 20th century (76). In addition, we did not include Chile, South Korea, and Croatia in the cointegration analysis due to limited data availability, spanning less than 20 y. All of the analyses were carried out with R software (77) and are fully reproducible, including data handling, from the public repository at https://zenodo.org/record/3571095.

Contributions to Mathematical Demography.
Changes over time in life expectancy. Changes over time in life expectancy at birth are a weighted average of rates of progress in reducing mortality (31). Letting $\ell(x, t)$ be the period life table probability at time t of surviving from birth to age x, life expectancy at birth can be expressed as follows:

$$e_o(t) = \int_0^\infty \ell(x, t) \, dx.$$

Because $\ell(x, t) = \exp[-\int_0^x \mu(a, t) \, da]$, where $\mu(a, t)$ is the force of mortality (hazard rate) at age a at time t, changes over time in $e_o(t)$ are given by the following:

$$\frac{\partial}{\partial t} e_o(t) = \dot{e}_o(t) = \int_0^\infty \frac{\partial}{\partial t} \ell(x, t) \, dx = -\int_0^\infty \ell(x, t) \int_0^x \frac{\partial}{\partial t} \mu(a, t) \, da \, dx$$

$$= -\int_0^\infty \frac{\partial}{\partial t} \mu(x, t) \int_x^\infty \ell(a, t) \, da \, dx.$$

[1]

A dot over a function denotes its partial derivative with respect to time. For simplicity, variable t will be omitted as an argument in the following. We define the following:

$$\rho(x) = -\frac{\dot{\mu}(x)}{\mu(x)} \text{ and } e(x) = \frac{1}{\ell(x)} \int_x^\infty \ell(a)\mathrm{d}a,$$

as the age-specific rates of mortality improvement over time and the remaining life expectancy at age x, respectively. Then, Eq. **1** can be expressed in terms of these two functions as follows:

$$\dot{e}_o = \int_0^\infty \mu(x)\ell(x)e(x)\rho(x)\mathrm{d}x = \int_0^\infty w(x)\rho(x)\mathrm{d}x. \qquad [2]$$

This last result shows that changes over time in life expectancy at birth are a weighted total of rates of progress in reducing mortality, with weights given by the function $w(x) = \mu(x)\ell(x)e(x)$, as shown by Vaupel and Canudas-Romo (31).

Measures of life span equality and their change over time. Several indicators have been proposed to measure variation in age at death (27, 78, 79). Selecting the best measure when comparing aging patterns among populations that differ in length of life is of great importance, since indicators vary in their sensitivity to mortality fluctuations and in their mathematical interpretation (27). In this study, we use three indicators based on the pace and shape of aging framework (25), which suggests a set of properties that indicators should satisfy (26, 80).

A variant of the life table entropy: h. A measure of life span inequality is the life table entropy $\bar{H}$ (29, 62, 63), which can be defined as follows:

$$\bar{H} = \int_0^\infty c(x)H(x)\mathrm{d}x,$$

where $c(x) = \ell(x) / \int_x^\infty \ell(a)\mathrm{d}a$ is the life table age composition, and $H(x) = \int_0^x \mu(a)\mathrm{d}a$ is the cumulative hazard to age x. Hence, $\bar{H}$ can be interpreted as an average value of the cumulative hazard. It can also be expressed as follows:

$$\bar{H} = -\frac{\int_0^\infty \ell(x)\ln \ell(x)\mathrm{d}x}{\int_0^\infty \ell(x)\mathrm{d}x} = \frac{e^\dagger}{e_o},$$

where $e^\dagger = -\int_0^\infty \ell(x)\ln \ell(x)\mathrm{d}x$ accounts for "life disparity," the average number of life-years lost as a result of death or the average remaining life expectancy at ages of death (9). For instance, an individual dying at age 50 in a population with remaining life expectancy at age 50 of 20 y would have lost those 20 y of life.

This definition of entropy provides a dimensionless indicator of relative variation in the length of life compared to life expectancy at birth, permitting comparison of populations with different age-at-death distributions (26). An alternative measure to $\bar{H}$ is the following:

$$h = -\ln \bar{H} = \ln e_o - \ln e^\dagger, \qquad [3]$$

which has previously been used to study life span equality across different primate populations, including humans (11). Note that $\bar{H}$ can be interpreted as an indicator of "life span inequality," given that higher values represent more variation in life spans, whereas h (the logarithm of the inverse) is a measure of "life span equality." From Eq. **3**, the variation over time in h is given by the following:

$$\dot{h} = \frac{\dot{e}_o}{e_o} - \frac{\dot{e}^\dagger}{e^\dagger}. \qquad [4]$$

An equivalent expression to Eq. **4** was previously derived using calculus of variation by Fernandez and Beltrán-Sánchez (81), who found that

$$\dot{\bar{H}}/\bar{H} = \frac{\dot{e}^\dagger}{e^\dagger} - \frac{\dot{e}_o}{e_o}.$$

This shows that changes over time in h are equal to minus the relative change in the life table entropy $\bar{H}$. Similarly to life expectancy at birth, Aburto et al. (52) proved that

$$\dot{\bar{H}}/\bar{H} = \int_0^\infty w(x)W(x)\rho(x)\mathrm{d}x,$$

where $w(x) = \mu(x)\ell(x)e(x)$ are the same weights for changes over time in e_o defined in Eq. **2**, and

$$W(x) = \frac{1}{e^\dagger}\left(H(x) + \bar{H}(x) - 1\right) - \frac{1}{e_o}.$$

Function $\bar{H}(x) = e^\dagger(x)/e(x)$ is the entropy conditional on surviving to age x, where $e^\dagger(x)$ refers to life disparity above age x, and $e(x)$ is the remaining life expectancy at age x (52). Because $\dot{h} = -\dot{\bar{H}}/\bar{H}$, it follows that

$$\dot{h} = \int_0^\infty w(x)W_h(x)\rho(x)\mathrm{d}x, \qquad [5]$$

with $W_h(x) = -W(x)$. This result shows that changes in life span equality over time are weighted totals of rates of progress in reducing mortality $\rho(x)$, with weights given by the product $w(x)W_h(x)$.

A variant of the Gini coefficient: g. The Gini coefficient is a popular index in social science used to measure distributions of positive variables, such as income (82). It has also been used to describe inequality in life spans as a measure of population health and in survival analysis as an indicator of concentration in survival times (26, 28, 64, 83, 84). In life table notation, the Gini coefficient G is given by the following:

$$G = 1 - \frac{\int_0^\infty \ell(x)^2\mathrm{d}x}{\int_0^\infty \ell(x)\mathrm{d}x} = 1 - \frac{\vartheta}{e_o}. \qquad [6]$$

Function $\vartheta = \int_0^\infty \ell(x)^2\mathrm{d}x$ relates to perturbation theory as it measures life expectancy from doubling the risk of death at all ages. From Eq. **6**, G can also be expressed in terms of the life table age distribution,

$$G = 1 - \int_0^\infty c(x)\ell(x)\mathrm{d}x = 1 - \bar{\ell}.$$

Note that $\bar{\ell} = \vartheta/e_o = \int_0^\infty c(x)\ell(x)\mathrm{d}x$ is a dimensionless indicator of life span equality, bounded between 0 and 1. If life spans are completely concentrated, all individuals die at the same age, the indicator equals 1; if they are equally spread the indicator tends to 0. In addition, if two babies are born at the same time in a population, then $\bar{\ell}$ measures their shared life span as a proportion of life expectancy (85). An alternative indicator to the Gini coefficient is the logarithm of its inverse:

$$g = -\ln G = -\ln(1 - \bar{\ell}), \qquad [7]$$

which is also a measure of equality rather than inequality.

Note that the derivative of $\bar{\ell}$ with respect to time is as follows:

$$\dot{\bar{\ell}} = \frac{\vartheta \dot{e}_o - \vartheta \dot{e}_o}{e_o^2}.$$

Hence, changes over time in g are given by the following:

$$\dot{g} = \frac{-\dot{\bar{\ell}}}{1 - \bar{\ell}} = \frac{\dot{\vartheta}e_o - \vartheta\dot{e}_o}{e_o^2} \cdot \frac{e_o}{e_o - \vartheta} = \frac{\dot{\vartheta} - \vartheta\dot{e}_o/e_o}{e_o - \vartheta} = \frac{\vartheta}{e_o - \vartheta}\left(\frac{\dot{\vartheta}}{\vartheta} - \frac{\dot{e}_o}{e_o}\right). \qquad [8]$$

Similar to h, the time derivative of g can be reexpressed as follows:

$$\dot{g} = \int_0^\infty w(x)W_g(x)\rho(x)\mathrm{d}x, \qquad [9]$$

where $w(x) = \mu(x)\ell(x)e(x)$ are the same weights for changes over time in e_o, and

$$W_g(x) = \frac{\vartheta}{e_o - \vartheta}\left(\frac{2\ell(x)\bar{\ell}(x)}{\vartheta} - \frac{1}{e_o}\right).$$

Function $\bar{\ell}(x)$ is defined as follows:

$$\bar{\ell}(x) = \frac{1}{\ell(x)} \int_x^\infty c(a)\ell(a)\mathrm{d}a,$$

and can be interpreted as life span equality above age x. A detailed proof of Eq. **9** can be found in *SI Appendix*, section B. This result shows that changes in life span equality over time, measured by g, are a weighted total of the rates of progress in reducing mortality $\rho(x)$, with weights given by the product $w(x)W_g(x)$.

A variant of the coefficient of variation: v. The coefficient of variation of the age-at-death distribution is the quotient of its SD σ and the life expectancy at birth:

$$CV = \frac{\sigma}{e_o} = \frac{\sqrt{\int_0^\infty (x - e_o)^2 \ell(x) \mu(x) \mathrm{d}x}}{\int_0^\infty \ell(x) \mathrm{d}x}.$$

This indicator has been previously used to measure life span inequality (24, 26). Here, we define a measure of life span equality as the logarithm of the inverse of the coefficient of variation,

$$v = -\ln CV = \ln e_o - \ln \sigma. \qquad [10]$$

Similar to life table entropy and the Gini coefficient, changes over time in v are given by the following:

$$\dot{v} = \frac{\dot{e}_o}{e_o} - \frac{\dot{\sigma}}{\sigma}, \qquad [11]$$

which can be reexpressed as follows:

$$\dot{v} = \int_0^\infty w(x) W_v(x) \rho(x) \mathrm{d}x. \qquad [12]$$

As before, $w(x)$ are the weights for e_o, whereas $W_v(x)$ are weights defined as follows:

$$W_v(x) = \frac{1}{e_o} - \frac{1}{\sigma^2} CV(x),$$

where

$$CV(x) = \frac{\int_x^\infty \ell(a)(a - e_o) \mathrm{d}a}{\int_x^\infty \ell(a) \mathrm{d}a} = \int_x^\infty c(a) a \, \mathrm{d}a - e_o = \bar{a}_x - e_o.$$

Note that $CV(x)$ is a weighted average of deviations from life expectancy at age x, which can be expressed as the difference between the average age of the population above age x ($\bar{a}_x$) and the life expectancy at birth. A detailed proof of Eq. 12 can be found in *SI Appendix*, section C. This result shows that changes over time in the alternative measure v of the coefficient of variation are a weighted total of the rates of progress in reducing mortality $\rho(x)$, with weights given by the product $w(x)W_v(x)$.

Demographic Methods to Calculate Threshold Ages and Age-Specific Contributions. From life tables, we calculated for each of the three indicators the threshold age below which averting deaths increases life span equality, and above which equality decreases. Eqs. 5, 9, and 12 indicate that the age-specific contribution to changes over time in life span equality can be expressed as the product $\rho(x)w(x)W_k(x)$, for $k \in \{h, g, v\}$. Note that $w(x)$ is a strictly positive function, whereas the indicator-specific weights $W_k(x)$ are strictly decreasing. Hence, under the assumption that death rates remain constant or decline at all ages [i.e., $\rho(x) \geq 0$ for all x] or remain constant or increase at all ages [i.e., $\rho(x) \leq 0$ for all x], for each indicator there is unique threshold age that we denote by a^h, a^g, and a^v, respectively. These threshold ages are reached when the corresponding weights equal 0; that is, when $W_h(x) = 0$, $W_g(x) = 0$ or $W_v(x) = 0$. The assumption that death rates need to decline (or increase) at all ages is necessary to have a unique threshold age. If death rates increase for some ages and decline for others, there may be several threshold ages that separate positive from negative contributions to life span equality, since the product $\rho(x)w(x)W_k(x)$ may switch from positive

to negative several times across ages. For instance, whenever $\rho(x)$ and $W(x)$ are both positive (or both negative), contributions will be positive; on the contrary, whenever $W(x) > 0$ and $\rho(x) < 0$, or $W(x) < 0$ and $\rho(x) > 0$, contributions will be negative. We quantified age-specific contributions to yearly changes in life expectancy and life span equality for all of the data available and estimated contributions above and below those thresholds. We used a model defined on a continuous framework that assumes gradual change in mortality over time (86) used in previous studies of life span inequality (13, 20, 21, 24).

Stochastic Properties of Life Expectancy and Life Span Equality. We analyzed the stochastic properties of e_o and life span equality over time to determine whether they are stationary processes (for further details, see *SI Appendix*, section A). In case of nonstationarity, we also find the order of integration. We performed the Kwiatkowski–Phillips–Schmidt–Shin test (87) for e_o and the three measures of life span equality, and the augmented Dickey–Fuller test (88) in their levels and first differences, respectively (we also perform tests against higher orders of integration but could not reject the hypothesis that the variables were integrated at a lower level). Using the 95% critical values, the null hypothesis of stationarity can be rejected in 94.9% of the cases for life expectancy and 93.9% for life span equality h. Moreover, at the same level, the null hypothesis of a unit root in their first differences is rejected in 97% of the cases for e_o and h. These analyses suggest that the variables are nonstationary processes and achieve stationarity after differencing once for both females and males. In the statistical analysis, we treat both variables as integrated of order one. The concept of cointegration was developed to avoid misleading interpretations regarding the relationship between two integrated variables (89). It refers to the case of a model that can adjust for stochastic trends to produce stationary residuals, and it permits detection of stable long-run relationships among integrated variables. Formally, two cointegrated variables can be expressed using a two-dimensional vector autoregressive model in its error correction form, defined as follows:

$$\Delta z_t = \Gamma \Delta z_{t-1} + \alpha \beta' z_{t-1} + \mu + \varepsilon_t.$$

Operator Δ denotes the first differences; z_t is a 2×1 vector of stochastic variables (e_o and life span equality in our case) at time t; Γ contains the cumulative long-run impacts; α and β are two 2×1 vectors of full rank; μ is a vector of constants; and ε_t is a vector of normally, independently, and identically distributed errors with zero means and constant variances (90). We specify the model with an unrestricted constant in the cointegration space and dummy variables in contexts where life expectancy experienced historical shocks, such as world wars and epidemics (see *SI Appendix*, Table S2 and section A, for additional details and sensitivity analyses).

Data Availability. Description to access the data and the code to reproduce results are in a permanent repository, accessible via the following link: https://zenodo.org/record/3571095. All data are publicly available.

1. J. Oeppen, J. W. Vaupel, Broken limits to life expectancy. *Science* **296**, 1029–1031 (2002).
2. J. C. Riley, *Rising Life Expectancy: A Global History* (Cambridge University Press, 2001).
3. D. Kirk, Demographic transition theory. *Popul. Stud. (Camb.)* **50**, 361–387 (1996).
4. F. W. Notestein, "Population—the long view" in *Food for the World*, T. Schulz, Ed. (University of Chicago Press, Chicago, 1945), pp. 36–57.
5. University of California, Berkeley; Max Planck Institute for Demographic Research (Germany), Human mortality database. https://www.mortality.org/. Accessed 22 July 2019.
6. R. D. Edwards, Changes in world inequality in length of life: 1970–2000. *Popul. Dev. Rev.* **37**, 499–528 (2011).
7. R. D. Edwards, S. Tuljapurkar, Inequality in life spans and a new perspective on mortality convergence across industrialized countries. *Popul. Dev. Rev.* **31**, 645–674 (2005).
8. J. Smits, C. Monden, Length of life inequality around the globe. *Soc. Sci. Med.* **68**, 1114–1123 (2009).
9. J. W. Vaupel, Z. Zhang, A. A. van Raalte, Life expectancy and disparity: An international comparison of life table data. *BMJ Open* **1**, e000128 (2011).
10. J. R. Wilmoth, S. Horiuchi, Rectangularization revisited: Variability of age at death within human populations. *Demography* **36**, 475–495 (1999).

11. F. Colchero *et al.*, The emergence of longevous populations. *Proc. Natl. Acad. Sci. U.S.A.* **113**, E7681–E7690 (2016).
12. A. A. van Raalte, I. Sasson, P. Martikainen, The case for monitoring life-span inequality. *Science* **362**, 1002–1004 (2018).
13. J.-A. Alvarez, J. M. Aburto, V. Canudas-Romo, Latin American convergence and divergence towards the mortality profiles of developed countries. *Popul. Stud. (Camb.)* **74**, 75–92 (2020).
14. I. Sasson, Trends in life expectancy and lifespan variation by educational attainment: United States, 1990–2010. *Demography* **53**, 269–293 (2016).
15. A. A. van Raalte *et al.*, More variation in lifespan in lower educated groups: Evidence from 10 European countries. *Int. J. Epidemiol.* **40**, 1703–1714 (2011).
16. L. Németh, Life expectancy versus lifespan inequality: A smudge or a clear relationship? *PLoS One* **12**, e0185702 (2017).
17. V. M. Shkolnikov, E. M. Andreev, Z. Zhang, J. Oeppen, J. W. Vaupel, Losses of expected lifetime in the United States and other developed countries: Methods and empirical analyses. *Demography* **48**, 211–239 (2011).
18. R. Seaman, A. H. Leyland, F. Popham, Increasing inequality in age of death at shared levels of life expectancy: A comparative study of Scotland and England and Wales. *SSM Popul. Health* **2**, 724–731 (2016).

19. H. Brønnum-Hansen, Socially disparate trends in lifespan variation: A trend study on income and mortality based on nationwide Danish register data. *BMJ Open* **7**, e014489 (2017).

20. J. M. Aburto, H. Beltrán-Sánchez, Upsurge of homicides and its impact on life expectancy and life span inequality in Mexico, 2005–2015. *Am. J. Public Health* **109**, 483–489 (2019).

21. J. M. Aburto, A. van Raalte, Lifespan dispersion in times of life expectancy fluctuation: The case of Central and Eastern Europe. *Demography* **55**, 2071–2096 (2018).

22. J. García, J. M. Aburto, The impact of violence on Venezuelan life expectancy and lifespan inequality. *Int. J. Epidemiol.* **48**, 1593–1601 (2019).

23. B. Seligman, G. Greenberg, S. Tuljapurkar, Equity and length of lifespan are not the same. *Proc. Natl. Acad. Sci. U.S.A.* **113**, 8420–8423 (2016).

24. J. M. Aburto, M. Wensink, A. van Raalte, R. Lindahl-Jacobsen, Potential gains in life expectancy by reducing inequality of lifespans in Denmark: An international comparison and cause-of-death analysis. *BMC Public Health* **18**, 831 (2018).

25. A. Baudisch, The pace and shape of ageing. *Methods Ecol. Evol.* **2**, 375–382 (2011).

26. T. F. Wrycza, T. I. Missov, A. Baudisch, Quantifying the shape of aging. *PLoS One* **10**, e0119163 (2015).

27. A. A. van Raalte, H. Caswell, Perturbation analysis of indices of lifespan variability. *Demography* **50**, 1615–1640 (2013).

28. K. Hanada, A formula of Gini's concentration ratio and its application to life tables. *J. Jpn. Stat. Soc.* **13**, 95–98 (1983).

29. C. Leser, Variations in mortality and life expectation. *Popul. Stud.* **9**, 67–71 (1955).

30. S. Tuljapurkar, "The final inequality: Variance in age at death" in *Demography and the Economy*, J. B. Shoven, Ed. (University of Chicago Press, 2010), pp. 209–221.

31. J. W. Vaupel, V. C. Romo, Decomposing change in life expectancy: A bouquet of formulas in honor of Nathan Keyfitz's 90th birthday. *Demography* **40**, 201–216 (2003).

32. B. Dormont, A.-L. Samson, M. Fleurbaey, S. Luchini, E. Schokkaert, Individual uncertainty about longevity. *Demography* **55**, 1829–1854 (2018).

33. J. R. Brown, Private pensions, mortality risk, and the decision to annuitize. *J. Public Econ.* **82**, 29–62 (2001).

34. L. A. Scott-Sheldon, M. P. Carey, P. A. Vanable, T. E. Senn, Subjective life expectancy and health behaviors among STD clinic patients. *Am. J. Health Behav.* **34**, 349–361 (2010).

35. United Nations, "World population prospects 2019: Ten key findings" (Population Division, Department of Economic and Social Affairs, United Nations, 2019).

36. C. W. Granger, P. Newbold, Spurious regressions in econometrics. *J. Econom.* **2**, 111–120 (1974).

37. A. Barford, D. Dorling, G. Davey Smith, M. Shaw, Life expectancy: Women now on top everywhere. *BMJ* **332**, 808 (2006).

38. V. Canudas-Romo, *Decomposition Methods in Demography* (Rozenberg Publishers Amsterdam, 2003).

39. J. H. Pollard, On the decomposition of changes in expectation of life and differentials in life expectancy. *Demography* **25**, 265–276 (1988).

40. E. E. Arriaga, Measuring and explaining the change in life expectancies. *Demography* **21**, 83–96 (1984).

41. E. M. Andreev, V. M. Shkolnikov, A. Z. Begun, Algorithm for decomposition of differences between aggregate demographic measures and its application to life expectancies, healthy life expectancies, parity-progression ratios and total fertility rates. *Demogr. Res.* **7**, 499–522 (2002).

42. N. Keyfitz, *Introduction to the Mathematics of Population: With Revisions* (Addison-Wesley Series in Behavioral Science: Quantitative Methods, Addison-Wesley, 1977).

43. S. Mitra, A short note on the taeuber paradox. *Demography* **15**, 621–623 (1978).

44. L. Demetrius, Relations between demographic parameters. *Demography* **16**, 329–338 (1979).

45. N. Goldman, G. Lord, A new look at entropy and the life table. *Demography* **23**, 275–282 (1986).

46. J. W. Vaupel, How change in age-specific mortality affects life expectancy. *Popul. Stud. (Camb.)* **40**, 147–157 (1986).

47. E. M. Andreev, Metod komponent v analize prodoljitelnosty zjizni. [The method of components in the analysis of length of life]. *Vestn. Statistiki* **9**, 147 (1982).

48. R. Pressat, Contribution des écarts de mortalité par âge à la différence des vies moyennes. *Population (Paris)* **40**, 766–770 (1985) (French edition).

49. W. Zuo, S. Jiang, Z. Guo, M. W. Feldman, S. Tuljapurkar, Advancing front of old-age human survival. *Proc. Natl. Acad. Sci. U.S.A.* **115**, 11209–11214 (2018).

50. V. Canudas-Romo, Three measures of longevity: Time trends and record values. *Demography* **47**, 299–312 (2010).

51. R. Rau, E. Soroko, D. Jasilionis, J. W. Vaupel, Continued reductions in mortality at advanced ages. *Popul. Dev. Rev.* **34**, 747–768 (2008).

52. J. M. Aburto, J.-A. Alvarez, F. Villavicencio, J. W. Vaupel, The threshold age of the lifetable entropy. *Demogr. Res.* **41**, 83–102 (2019).

53. D. O. Gillespie, M. V. Trotter, S. D. Tuljapurkar, Divergence in age patterns of mortality change drives international divergence in lifespan inequality. *Demography* **51**, 1003–1017 (2014).

54. Z. Zhang, J. W. Vaupel, The age separating early deaths from late deaths. *Demogr. Res.* **20**, 721–730 (2009).

55. J.-M. Robine, Redefining the stages of the epidemiological transition by a study of the dispersion of life spans: The case of France. *Popul. Engl. Selection* **13**, 173–193 (2001).

56. J. P. Mackenbach, The epidemiologic transition theory. *J. Epidemiol. Community Health* **48**, 329–331 (1994).

57. V. Zarulli et al., Women live longer than men even during severe famines and epidemics. *Proc. Natl. Acad. Sci. U.S.A.* **115**, E832–E840 (2018).

58. M. Marmot, R. Wilkinson, *Social Determinants of Health* (OUP Oxford, 2005).

59. E. Nolte, M. McKee, *Does Health Care Save Lives? Avoidable Mortality Revisited* (The Nuffield Trust, 2004).

60. E. Nolte, M. McKee, Variations in amenable mortality—trends in 16 high-income nations. *Health Policy* **103**, 47–52 (2011).

61. W. H. Organization, "The world health report 2000: Health systems: Improving performance" (World Health Organization, 2000).

62. L. Demetrius, Adaptive value, entropy and survivorship curves. *Nature* **275**, 213–214 (1978).

63. N. Keyfitz, A. Golini, Mortality comparisons: The male-female ratio. *Genus* **31**, 1–34 (1975).

64. V. M. Shkolnikov, E. E. Andreev, A. Z. Begun, Gini coefficient as a life table function: Computation from discrete data, decomposition of differences and empirical examples. *Demogr. Res.* **8**, 305–358 (2003).

65. D. Jasilionis, F. Meslé, V. M. Shkolnikov, J. Vallin, Recent life expectancy divergence in Baltic countries. *Eur. J. Popul.* **27**, 403–431 (2011).

66. J. Rehm et al., Alcohol accounts for a high proportion of premature mortality in Central and Eastern Europe. *Int. J. Epidemiol.* **36**, 458–467 (2007).

67. S. L. K. Cheung, J.-M. Robine, E. J.-C. Tu, G. Caselli, Three dimensions of the survival curve: Horizontalization, verticalization, and longevity extension. *Demography* **42**, 243–258 (2005).

68. M. Dribe, M. Olsson, P. Svensson, *Famines in the Nordic Countries, AD 536–1875* (Lund Papers in Economic History, Department of Economic History, Lund University, 2015), vol. 138.

69. J. Vaupel, H. Lundstrom, "Longer life expectancy? Evidence from Sweden of reductions in mortality rates at advanced ages" in *Studies in the Economics of Aging* (University of Chicago Press, 1994), pp. 79–102.

70. J. W. Vaupel et al., Biodemographic trajectories of longevity. *Science* **280**, 855–860 (1998).

71. O. R. Jones et al., Diversity of ageing across the tree of life. *Nature* **505**, 169–173 (2014).

72. A. Baudisch et al., The pace and shape of senescence in angiosperms. *J. Ecol.* **101**, 596–606 (2013).

73. N. Stroustrup et al., The temporal scaling of *Caenorhabditis elegans* ageing. *Nature* **530**, 103–107 (2016).

74. C. Ruth Archer et al., Diet has independent effects on the pace and shape of aging in Drosophila melanogaster. *Biogerontology* **19**, 1–12 (2018).

75. S. H. Preston, P. Heuveline, M. Guillot, *Demography. Measuring and Modeling Population Processes* (Blackwell, 2001).

76. K. Christensen, G. Doblhammer, R. Rau, J. W. Vaupel, Ageing populations: The challenges ahead. *Lancet* **374**, 1196–1208 (2009).

77. R Core Team, R: A Language and Environment for Statistical Computing (R Foundation for Statistical Computing, Vienna, Austria, 2019).

78. D. G. Goodwin, J. W. Vaupel, "Concentration curves and have-statistics for ecological analysis of diversity" in *Part I: Dominance and Evenness in Reproductive Success* (International Institute for Applied Systems Analysis, Laxenburg, Austria, 1985), pp. WP-85–WP-72.

79. A. Wagstaff, P. Paci, E. van Doorslaer, On the measurement of inequalities in health. *Soc. Sci. Med.* **33**, 545–557 (1991).

80. T. F. Wrycza, A. Baudisch, The pace of aging: Intrinsic time scales in demography. *Demogr. Res.* **30**, 1571–1590 (2014).

81. O. E. Fernandez, H. Beltrán-Sánchez, The entropy of the life table: A reappraisal. *Theor. Popul. Biol.* **104**, 26–45 (2015).

82. C. Gini, "Variabilità e mutabilità" in *Memorie di Metodologica Statistica*, E. Pizetti, T. Salvemini, Eds. (Libreria Eredi Virgilio Veschi, Rome, 1912), vol. 1, pp. 211–382.

83. C. Gigliarano, U. Basellini, M. Bonetti, Longevity and concentration in survival times: The log-scale-location family of failure time models. *Lifetime Data Anal.* **23**, 254–274 (2017).

84. M. Bonetti, C. Gigliarano, P. Muliere, The Gini concentration test for survival data. *Lifetime Data Anal.* **15**, 493–518 (2009).

85. J. A. Barthold Jones, A. Lenart, A. Baudisch, Complexity of the relationship between life expectancy and overlap of lifespans. *PLoS One* **13**, e0197985 (2018).

86. S. Horiuchi, J. R. Wilmoth, S. D. Pletcher, A decomposition method based on a model of continuous change. *Demography* **45**, 785–801 (2008).

87. D. Kwiatkowski, P. C. Phillips, P. Schmidt, Y. Shin, Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *J. Econom.* **54**, 159–178 (1992).

88. D. A. Dickey, W. A. Fuller, Distribution of the estimators for autoregressive time series with a unit root. *J. Am. Stat. Assoc.* **74**, 427–431 (1979).

89. C. W. Granger, Some properties of time series data and their use in econometric model specification. *J. Econom.* **16**, 121–130 (1981).

90. B. Pfaff, *Analysis of Integrated and Cointegrated Time Series with R* (Springer Science and Business Media, 2008).

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