THE ESTIMATION OF FERTILITY FROM INCOMPLETE COHORT DATA BY MEANS OF THE TRANSFORMED GOMPERTZ MODEL

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Heather Booth

London School of Hygiene and Tropical Medicine
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ABSTRACT

Among the several models of fertility that have been developed, the Gompertz function has been shown to perform quite well. The fit to cumulative fertility by age of women is better in the middle age range than in the tails of the distribution, especially for high fertility populations. An empirical transformation of the age scale is developed to produce a better fit to the entire reproductive period. The substitution of age by this transformed scale results in the transformed Gompertz model of fertility:

$$F(x) = FP^{Q^{Y_s(x)}} \qquad \begin{array}{l} 0 < P,\ Q < 1 \\ 0 < F \end{array}$$

where $F(x)$ is cumulative fertility by age of women, x, F is completed fertility, P and Q are parameters and $Y_s(x)$ is the transformed age scale. The model can also be written as:

$$Y(x) = \alpha + \beta Y_s(x)$$

where $Y(x) = -\ln(-\ln F(x)/F)$, $\alpha = -\ln(-\ln P)$ and $\beta = -\ln Q$. $Y_s(x)$ is obtained by averaging over a selection of transformed patterns of high fertility schedules generated by the empirically based Coale-Trussell model of fertility.

The model is tested using several sets of good quality fertility rates for birth cohorts of women, including a series of simulated rates developed especially for this purpose. Both the goodness of fit of the model and its projection capabilities from incomplete cohort data are shown to be good. Fitting is by the least squares method with equal weights.

Several sets of poorer quality, high fertility, cohort data obtained from maternity histories collected in Bangladesh, Sri Lanka and West New Guinea are used to illustrate the application of the model. Fitting for these data is by least squares with an infinite weight on reported parity at the time of the survey. The estimates of completed fertility are plausible, and the fitted curves provide evidence of reporting errors in the data. There is also evidence of trends over time in the level and pattern of fertility.

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In memory of

Joan

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CHAPTER 1

INTRODUCTION

During the last fifty years, a considerable amount of demographic research has been focused on the representation of fertility by means of various models. The majority has been concerned with models of fertility by age of women, but some has considered fertility by duration of marriage. In most, if not all, instances the natural time scale has been used.

The models have been used for two purposes: for the graduation of fertility rates and for the prediction of the level of fertility from incomplete data. Both period and cohort fertility rates have been used as illustrations.

Theoretical Models Applied to Fertility

Pearsonian Type I (Beta) and Type III (Gamma) curves were the first to be fitted to age specific fertility rates (Wicksell, 1931), followed in later years by proposals of functions by Hadwiger (1940, 1941) and Mazur (1963). These functions and the estimation of their parameters have been investigated by Tekse (1967), amongst others, but none has been shown to be consistently superior for all fertility patterns and all leave room for improvement.

The Pearson Type I function has received more recent attention (Mitra, 1967; Mitra and Romaniuk, 1973; Romaniuk, 1973). By assuming a fixed age interval of fertility, the number of parameters to be estimated is reduced from five to three without a significant loss of fit.

For representing the age pattern of fertility, Talwar (1970, 1974) expresses his preference for the S_B-system of curves because of their simplicity in fitting. This system is based on the translation of the data to the Normal distribution, and deals with ratios rather than rates so that the level parameter is lost.

The three parameter Gompertz function has also been successfully used to represent fertility rates. Martin (1967) used this function for the graduation of cohort data cumulated by age, as did Wunsch (1966). Romaniuk and Tanny (1969) fitted Gompertz curves to incomplete cohort data for prediction purposes, but found the results disappointing. A much improved iterative method of fitting was proposed by Murphy and Nagnur (1972), but reservations about the use of the curve for prediction were not dispelled. Their attempt to improve the fit in the tails of the distribution, by adding a fourth parameter to produce the Makeham curve, was also rejected on the grounds of loss of both compactness and an asymptotic approach to the level parameter value.

Work by Farid (1973) has shown that the Gompertz function can also be applied to the graduation of fertility by marriage duration. Very recently, Little (1978) has applied the model to period fertility by age using maximum likelihood estimation of the parameter.

Empirical Models of Fertility

Other work has concentrated on the development of fertility models based on empirical methods. Brass (1968, 1975) has developed a third degree polynomial for describing age specific fertility. The function is better suited to graduation within age intervals, rather than along the entire reproductive age span, and is used for this purpose in the P/F technique of fertility level estimation.

The Coale-Trussell model of fertility (Coale and Trussell, 1974) represents age-specific fertility rates as the product of two functions; one representing marriage patterns by age and the other representing marital fertility. Both of these functions are based on (historical) data so that the fits obtained might be expected to be better than those to theoretical functions. The model has the disadvantage of having four parameters, however.

The analysis of maternity history data has featured relatively recently, and methods for doing so have been empirical. This is largely due to the paucity, until recently, of good quality data of this type. Data collected in maternity history form are invaluable in that they permit an analysis of trends over time without the problems of differential biases and errors arising from successive surveys. The sample sizes required for such analysis are large because of the need to minimise sampling errors, particularly where subgroups, including age cohorts of women, are involved.

Brass (1971) has used maternity history data for West New Guinea to point out the problems involved in determining trends in fertility and to suggest methods of overcoming these problems to some extent. First births are used to correct the time distortions in all births by adopting as an internal standard the distribution of first births for the most recent time period. The pattern of deviation of first births from this standard for each cohort is taken as equal to the pattern of deviations of reported all births to corrected all births. Even if this assumption is valid, the method suffers from being restricted in its age range to that for first births.

Potter (1977) has adopted a completely different approach to the analysis of maternity history data. He has constructed a model of event misplacement based on the propositions that the more recent an event the more accurately it is reported, and that reports of events subsequent to the first report are not independent because intervals between events are considered by the respondent. Potter demonstrates that event misreporting of the type specified in his model leads to an apparent or overestimated decline in fertility, and presents evidence of such spurious declines.

The model developed in later chapters of this work differs from all of the above in that the natural time scale is replaced by a transformation of time. The original ideas for the development of such a model are those of Brass (1974, 1977). The particular transformation developed here is for use with data by age of women for high fertility populations. It is perfectly possible, however, to use the same methods to develop transformations for marriage duration data and for lower levels, and therefore different patterns, of fertility.

The transformation of the age scale is empirical rather than functional, and in fact represents a pattern of fertility typical of high fertility populations. The transformed Gompertz model, which is equivalent to the Gompertz model with the natural age scale replaced by the transformed, is thus a relational model in that observed fertility is related to a typical or standard pattern of fertility, rather than to age. For good quality data, the use of an internal standard might be envisaged; this possibility has been used for historical data by Petrioli (1975).

A full explanation of the transformed Gompertz model is given in Chapter 2. Using the Gompertz model as a basis, it is shown how the introduction of an appropriate transformation of the age scale improves the fit of the model. The change in the demographic interpretation of the parameters brought about by the transformation is also discussed. The actual empirical development of the transformation of the age scale is described in Chapter 3.

The development of a set of simulated data on which to test the model is the subject of Chapter 4. This involved the modification of an existing simulation model of fertility, and the specification of the various parameters of the model to produce a series of schedules representing a fertility decline. This simulated set of data and three sets of historical data were used to test the transformed Gompertz model, as discussed in Chapter 5.

The application of the model to data collected in maternity histories is presented in Chapter 6, whilst Chapter 7 contains the conclusions and some suggestions for further work.

CHAPTER 2

THE DEVELOPMENT OF THE TRANSFORMED GOMPERTZ MODEL

Introduction

Cumulative fertility has been represented by the (ordinary) Gompertz function on several occasions. It has been shown that the curve fits better in the middle of the childbearing distribution than in the tails. This chapter describes the Gompertz function and the reasons for its lack of fit at young and old ages, and shows how the function can be modified to produce better fits. The resulting model is the transformed Gompertz model of fertility.

The Gompertz Curve

The Gompertz function has been used to represent fertility by age of women. The function has the form

$$F(x) = F\,A^{B^{x-x_o}} \qquad 2.1$$

where x_o is an arbitrary origin of the age scale, and F(x) is the cumulative fertility distribution by age. The parameters A and B lie between 0 and 1, while the only restriction on F is that it should be positive.

F is the asymptote to which $F(x)$ tends as x increases, and therefore can be interpreted as the total fertility rate or completed fertility. A and B describe the shape of the fertility curve over age. A is the proportion of total fertility that is attained by age x_o, shown by evaluating equation 2.1 at $x = x_o$:

$$F(x_o) = FA^{B^o} = FA$$

and is therefore related to the location of the curve on the age scale. A change in the origin thus leads to a change in the parameter A, whilst B remains the same. If the origin is moved to $x_o + h$, the function can be redefined

$$F(x) = F\,S^{T^{x-x_o-h}} = F\,S^{T^{x-x_o}T^{-h}}$$

$$= F(S^{T^{-h}})^{T^{x-x_o}}$$

Comparison with equation 2.1 gives $A = S^{T^{-h}}$ and $B = T$.

Interpretation of B is not so straightforward. For given A, however, it can be shown that B is related to the variance of the distribution in that B increases with the variance. A more rigorous explanation, which depends on the properties of the mode of the first derivative of the Gompertz function (see Appendix 2.1), is given in Appendix 2.2.

The lack of fit of the Gompertz function

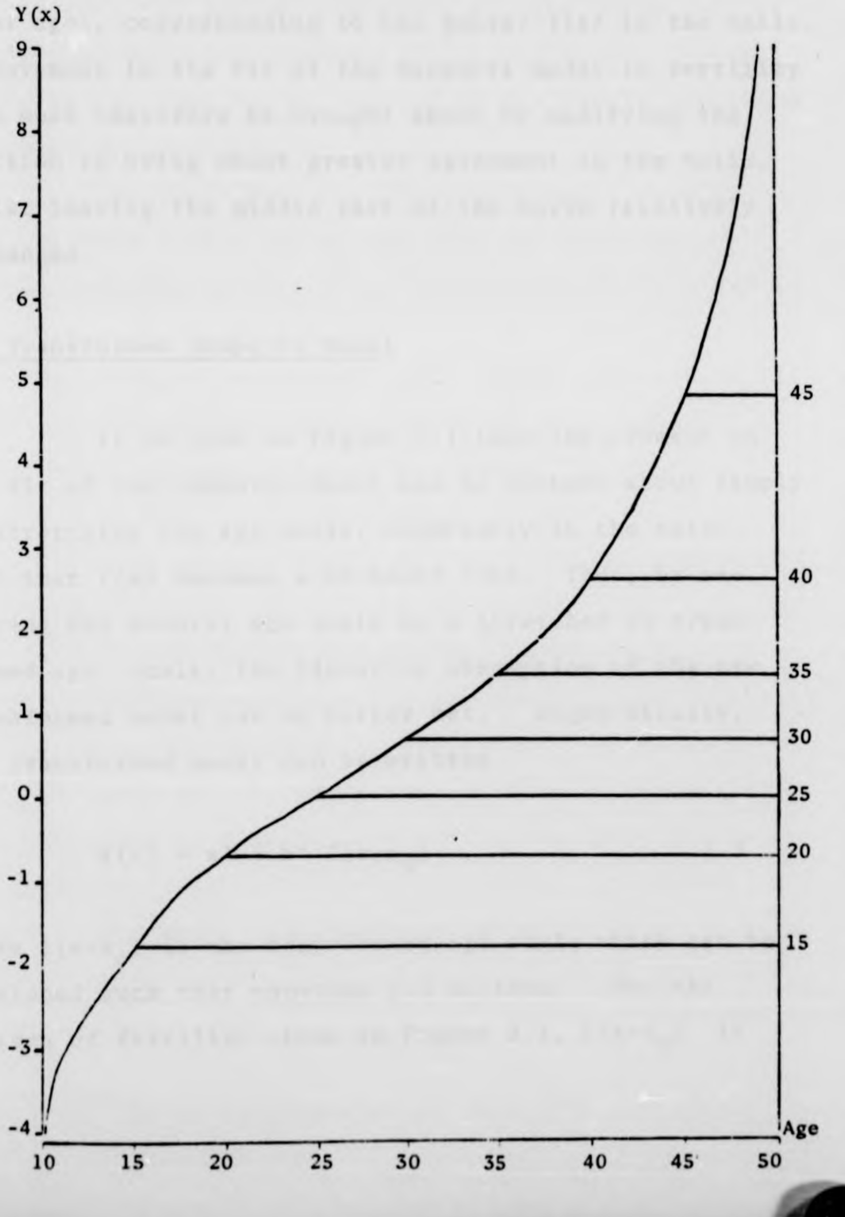
In describing cumulative fertility, the Gompertz function implicitly assumes that the double logarithm of fertility is linearly related to age. To take natural logarithms twice of equation 2.1 requires first that the expression be divided by F so that $F(x) \div F$ has an upper limit of 1, and secondly that a negative sign be introduced between taking the two logarithms to produce positive values for the second operation. Finally, the negative of the double logarithm is taken so that the coefficient, b, which is related to the variance, is positive. Hence

$$\begin{aligned} Y(x) &= -\ln(-\ln\frac{F(x)}{F}) = -\ln(-\ln A) + (-\ln B)(x-x_0) \\ &= a + b(x-x_0) \qquad 2.2 \end{aligned}$$

where $a = -\ln(-\ln A)$ and $b = -\ln B$, $-\infty < a < \infty$ and $0 < b < \infty$.

This assumption of linearity is not true, as is clearly seen in Figure 2.1 which shows transformed (that is the double logarithm) cumulative fertility, Y(x), against x for a pattern of fertility typical of high fertility populations. (In fact, this is the pattern of fertility developed in Chapter 3 as the standard.) The pattern of deviation of Y(x) from linearity gives some indication of how observed fertility

Figure 2.1: Transformed cumulative fertility against age for the empirically based standard.



deviates from the Gompertz model. It is seen that $Y(x)$ is almost linear at ages 20 to 35, corresponding to the better fit of the model at these ages, but deviates from linearity outside of this age range, most notably at older ages, corresponding to the poorer fits in the tails. Improvement in the fit of the Gompertz model to fertility data must therefore be brought about by modifying the function to bring about greater agreement in the tails, whilst leaving the middle part of the curve relatively unchanged.

The Transformed Gompertz Model

It is seen in Figure 2.1 that improvement in the fit of the Gompertz model can be brought about simply by stretching the age scale, especially in the tails, such that $Y(x)$ becomes a straight line. Thus, by replacing the natural age scale by a stretched or transformed age scale, the linearity assumption of the new transformed model can be better met. Algebraically, the transformed model can be written

$$Y(x) = a' + b' Z(x-x_o) \qquad 2.3$$

where $Z(x-x_o)$ is the transformed age scale which can be developed such that equation 2.3 is true. For the pattern of fertility shown in Figure 2.1, $Z(x-x_o)$ is equal to the points labelled 15, 20 ... 45 years on the right hand axis, with 10 years at $-\infty$ and 50 years at $+\infty$. The distances between these points are the amounts by which to stretch the age scale. At the same time, it is seen that the points themselves are the values of the transformed fertility pattern, $Y(x)$. The transformed age scale can thus be represented by the double logarithm of cumulative fertility.

The application of the transformed Gompertz model to data from a variety of high fertility populations requires the development of a typical or standard transformation of the age scale. This is equivalent to the development of a standard pattern of fertility, $F_s(x)$. The double logarithm transformation of this standard, $Y_s(x)$, then replaces $Z(x-x_0)$ in equation 2.3 giving

$$Y(x) = \alpha + \beta\, Y_s(x) \qquad 2.4$$

Transformed fertility is now assumed to be linearly related to transformed standard fertility, which is itself equivalent to the transformed age scale. In other words, it is assumed that the deviations of observed fertility from the Gompertz function are similar to those of the standard.

The model can also be written

$$F(x) = F\, P^{Q^{Y_s(x)}} \qquad 2.5$$

where F is the level parameter as before ($F > 0$), and

P and Q are new parameters describing the pattern of fertility where $0 < P, Q < 1$. The parameters, α and β, are related to P and Q such that $\alpha = -\ln(-\ln P)$ and $\beta = -\ln Q$, with $-\infty < \alpha < \infty$ and $0 < \beta < \infty$.

Demographic interpretation of the parameters of the transformed Gompertz model

Of the three parameters of the transformed Gompertz model, only the level parameter, F, retains the same interpretation as in the (ordinary) Gompertz model, namely that of total or completed fertility. The remaining two parameters describe the pattern of fertility, as in the Gompertz, but their exact interpretation is changed. Rather than relate to the natural age scale, the new parameters describe fertility in relation to the transformed age scale, that is to the pattern of standard fertility.

From equation 2.5 it is seen that at $Y_s(x) = 0$

$$F(x) = FP^{Q^0} = FP$$

P is therefore the proportion of fertility achieved by the age at which $Y_s(x) = 0$. This age can be regarded as the origin of the standard and will be denoted x_{os}. At this origin $F_s(x_{os}) = e^{-1}$, so that comparison of P with e^{-1} indicates the relative proportions of observed and standard fertility achieved by age x_{os}. When $P = e^{-1}$, the same proportion, e^{-1}, of fertility is achieved by x_{os} in both fertility schedules. Moreover, $P = e^{-1}$ indicates, by definition, that the origin of observed fertility, x_o, occurs at the same age as in the standard, that is $x_o = x_{os}$. This equivalence also implies that $\alpha = 0$.

Inequalities between P and e^{-1}, or between α and 0, indicate unequal proportions of observed and standard fertility achieved by age x_{os}. Values of P of less than e^{-1}, that is $\alpha < 0$, indicate that a smaller proportion of observed fertility is achieved by x_{os} than in the standard. The exact proportion is, of course, P. It follows that the origin of observed fertility, the age by which e^{-1} of fertility is attained, is later than x_{os}. Observed fertility is thus generally later than standard fertility.

In the converse situation where $P > e^{-1}$, and $\alpha > 0$, observed fertility is generally earlier than standard such that by age x_{os} a greater proportion of observed fertility is achieved. The origin of observed fertility is correspondingly earlier than x_{os}.

This interpretation of the parameter P (or α) of the transformed Gompertz model is illustrated in a series of diagrams contained in Figures 2.2 to 2.5 (where the Y-axes intersect at $Y(x_{os}) = Y_s(x_{os})$). Diagrams 1, 4 and 7 illustrate the case where $\alpha = 0$; diagrams 2, 5 standard fertility achieved by age x_{os}. When $P = e^{-1}$, the same proportion, e^{-1}, of fertility is achieved by x_{os} in both fertility schedules. Moreover, $P = e^{-1}$ indicates, by definition, that the origin of observed fertility, x_o, occurs at the same age as in the standard, that is $x_o = x_{os}$. This equivalence also implies that $\alpha = 0$.

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Figure 2.2: Relationships between observed and standard transformed fertility implied by α and β values: $Y(x)$ against $Y_s(x)$.

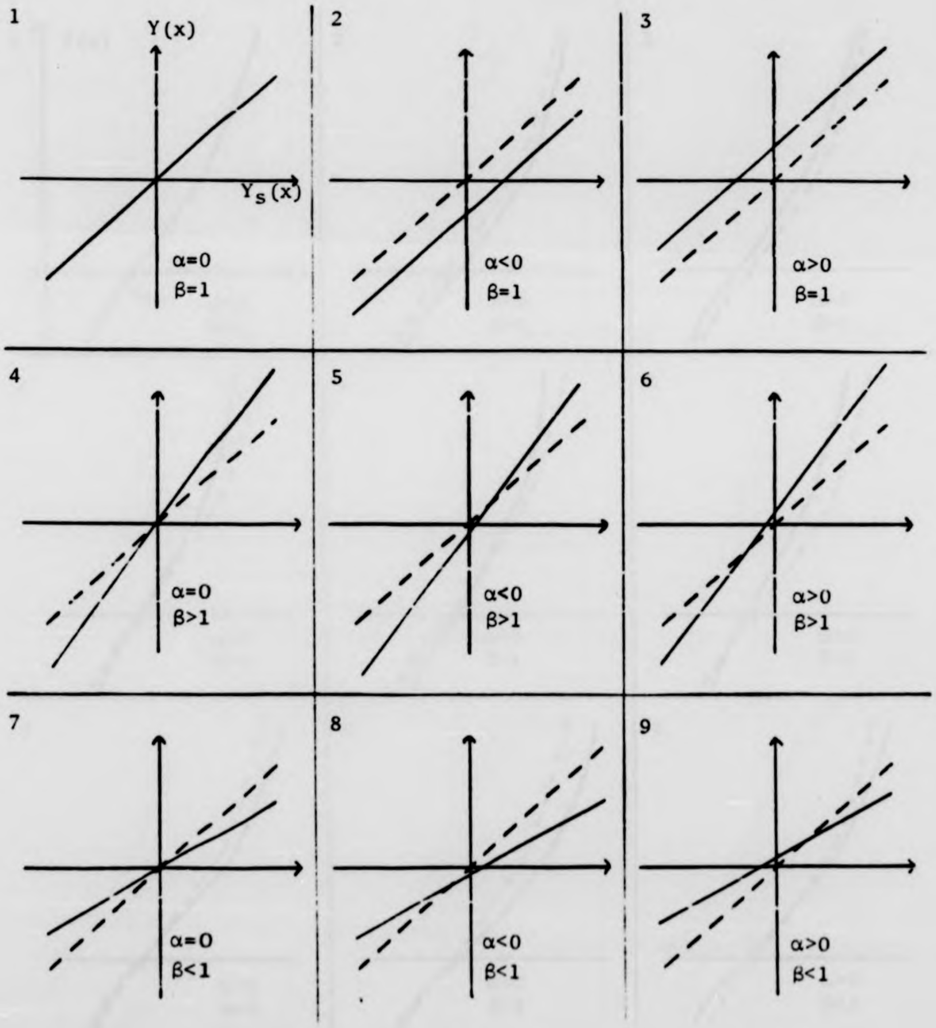


Figure 2.3: Relationships between observed and standard transformed fertility implied by α and β values: $Y(x)$ and $Y_s(x)$ against x.

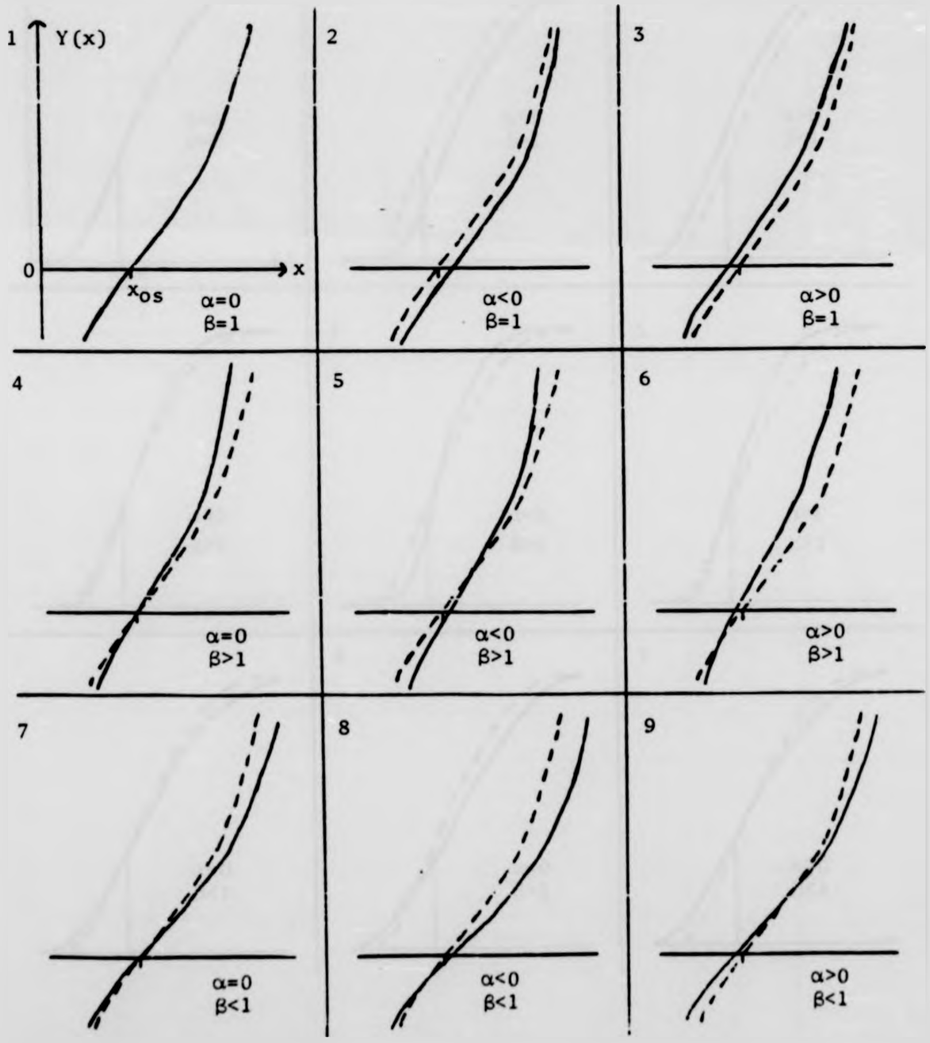


Figure 2.4: Relationships between observed and standard cumulative fertility implied by α and β values.

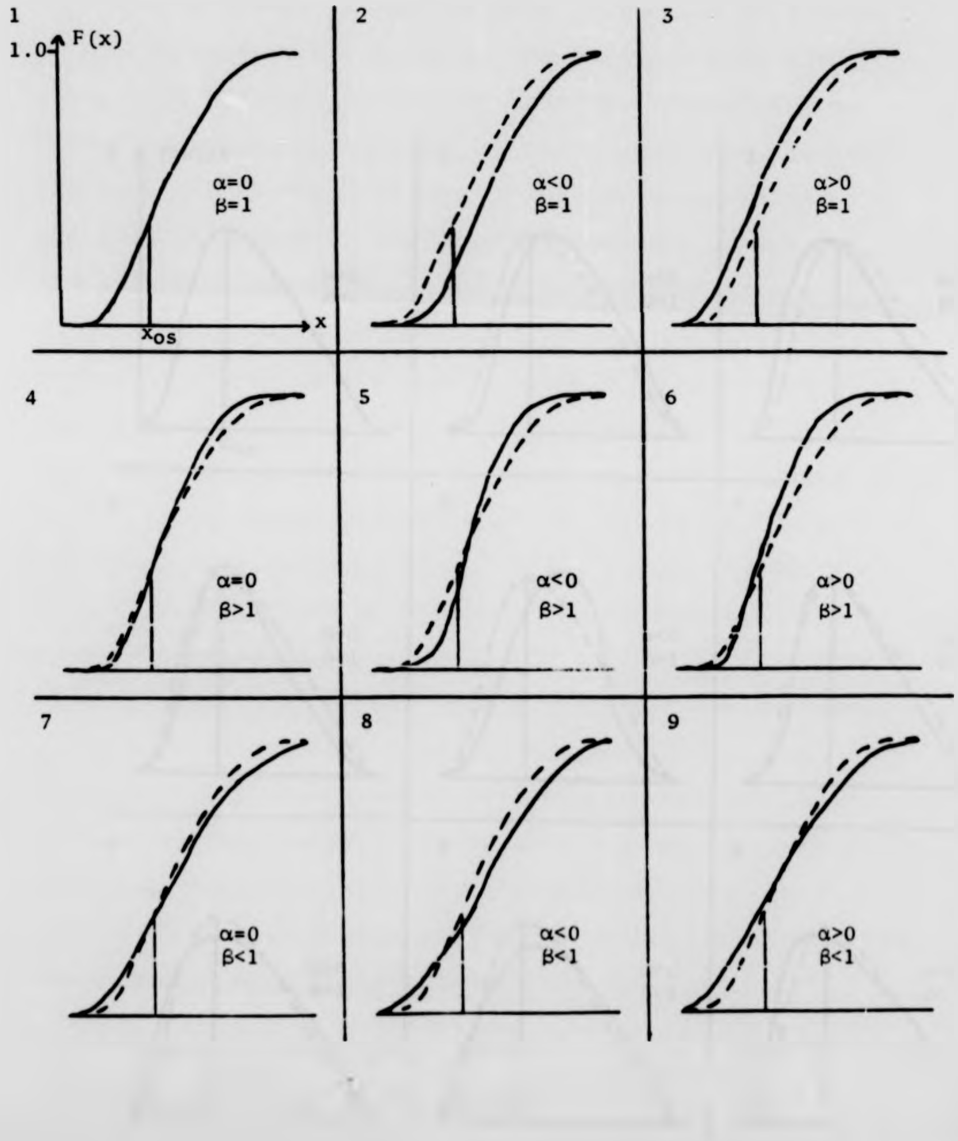
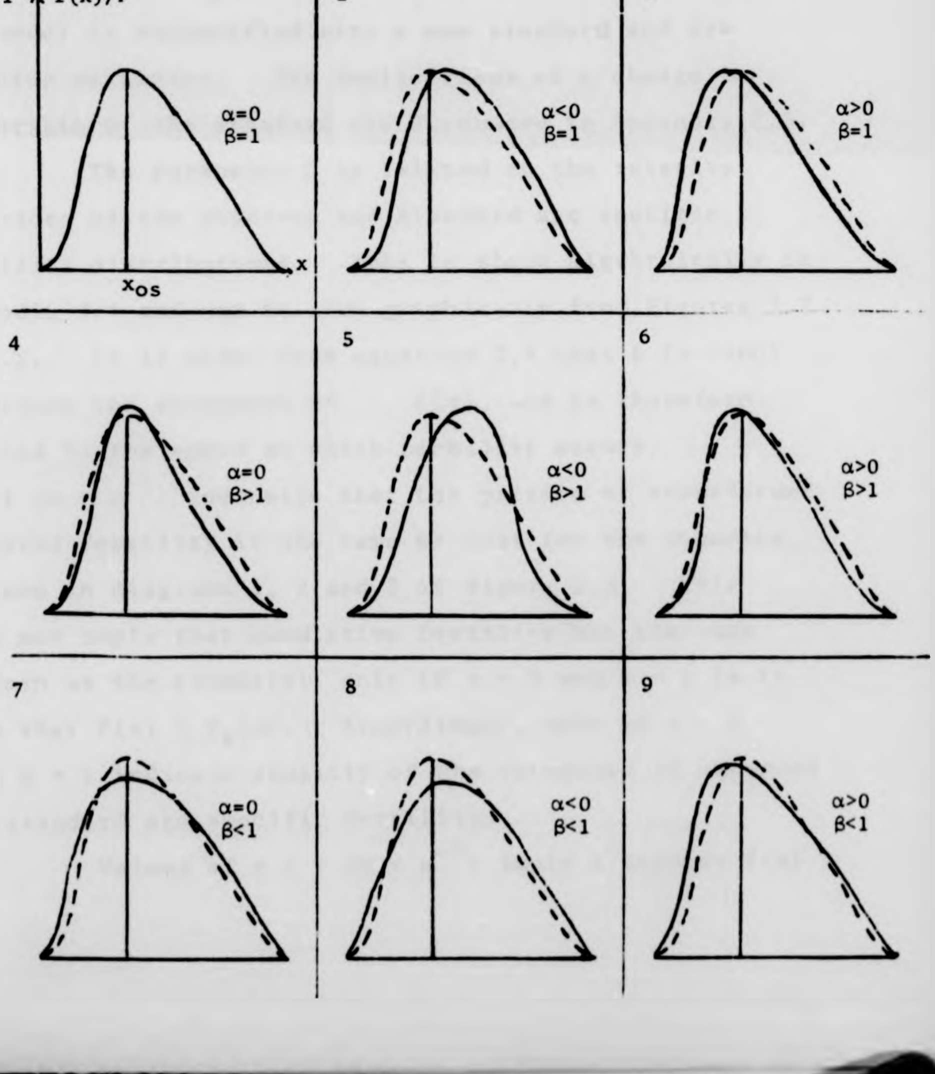


Figure 2.5: Relationships between observed and standard age specific fertility implied by α and β values.



and 8 show $\alpha < 0$; and diagrams 3, 6 and 9 show $\alpha > 0$. It is seen that α is related to the location of observed fertility in relation to the standard, as measured at the origin of the standard.

A change in the standard origin has no effect on the above interpretation of the location parameter. Rather the standard pattern of cumulative fertility is changed, because $F_s(x) = e^{-1}$ at a different age, so that the model is respecified with a new standard and new location parameter. The implications of a change in the origin of the standard are discussed in Appendix 2.3.

The parameter Q is related to the relative variances of the observed and standard age specific fertility distributions. This is shown algebraically in Appendix 2.4 and can be seen graphically from Figures 2.2 to 2.5. It is clear from equation 2.4 that β ($= -\ln Q$) describes the steepness of Y(x), and is therefore related to the speed at which fertility occurs. $\beta = 1$ ($Q = e^{-1}$) indicates that the pattern of transformed observed fertility is the same as that for the standard, as seen in diagrams 1, 2 and 3 of Figure 2.3. This does not imply that cumulative fertility has the same pattern as the standard: only if $\alpha = 0$ and $\beta = 1$ is it true that $F(x) = F_s(x)$. Accordingly, only if $\alpha = 0$ does $\beta = 1$ indicate equality of the variances of observed and standard age specific fertility.

Values of $\beta > 1$ ($Q < e^{-1}$) imply a steeper Y(x) than for the standard as seen in diagrams 4, 5 and 6 of Figures 2.2 and 2.3. In this situation, $Y(x)$ approaches its asymptotes of $-\infty$ at age 10 and $+\infty$ at age 50 more slowly than the standard (as seen in Figure 2.3). If $\beta = 1 + c$, where c is positive, then equation 2.4 becomes

$$\begin{aligned} Y(x) &= \alpha + (1+c)\ Y_s(x) \\ &= \alpha + Y_s(x) + cY_s(x) \qquad 2.6 \end{aligned}$$

The first two terms account for the portion of $Y(x)$ occurring when $\beta = 1$; the last term measures the amount of $Y(x)$ due to the excess of β over 1. For $Y_s(x) > 0$ this term is positive, and for $Y_s(x) < 0$ it is negative, so that $Y(x)$ approaches its asymptotes more slowly than when $\beta = 1$ for given α. On conversion to cumulative fertility the situation changes to one where $F(x)$ approaches its asymptotes more quickly than when $\beta = 1$. From equation 2.6

$$\begin{aligned} F(x) &= F\ e^{-e^{-\alpha-Y_s(x)-cY_s(x)}} \\ &= F\ P^{e^{-Y_s(x)}\ e^{-cY_s(x)}} \\ &= F[P^{e^{-Y_s(x)}}]^{e^{-cY_s(x)}} \qquad 2.7 \end{aligned}$$

The expression inside the square brackets corresponds to $\beta = 1$. The effect of β being greater than 1 is to raise this expression, which has a value between 0 and 1, to a positive power, $e^{-cY_s(x)}$. For $c > 0$, this means that for $Y_s(x) < 0$, $F(x)$ is less than is the case when $\beta = 1$, and for $Y_s(x) > 0$, $F(x)$ is greater than when $\beta = 1$. In other words, $F(x)$ approaches its asymptotes of zero and F more quickly than when $\beta = 1$ for given α. This is seen in diagrams 4, 5 and 6 of Figure 2.4. On translation to age specific fertility rates it becomes apparent that $\beta > 1$ implies a smaller variance for given α. This is seen in diagrams 4, 5 and 6 of Figure 2.5.

When $\beta < 1$ $(Q > e^{-1})$, $Y(x)$ is less steep than $Y_s(x)$ and approaches its asymptotes more quickly because c is negative in equation 2.6. Correspondingly, $F(x)$ approaches its asymptotes more slowly than when $\beta = 1$, again because of the negative value of c in equation 2.7. The variance of age specific fertility is therefore larger for given α than when $\beta = 1$. These relationships are shown in diagrams 7, 8 and 9 of Figures 2.2 to 2.5.

CHAPTER 3

STANDARD FERTILITY

Introduction

The development of a standard pattern of fertility for use in the transformed Gompertz model is the subject of this chapter. The standard is based on the Coale-Trussell model of fertility, itself based on empirical data. A description of this model is given below together with a detailed account of its use in the development of the standard.

The Coale-Trussell Model Fertility Schedules

The set of model fertility schedules developed by Coale and Trussell (1974) is an attempt to create a family of schedules encompassing the full range of human experience. The model is based on two functions: model proportions ever married by age and model marital fertility. The product of these two functions is assumed to describe the age pattern of fertility.

Proportions ever-married by age

Coale (1971) showed that first marriage frequencies

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Proportions ever-married by age

Coale (1971) showed that first marriage frequencies

(of cohorts) conform to a common underlying pattern, and that differences between populations (cohorts) can be attributed to differing ages at which first marriage begins coupled with the length of time taken for most marriages to occur. There is also a level factor, describing differences between populations in the final proportion ever married. This common underlying pattern or standard was based on first marriage frequencies recorded in Sweden in 1865-69. Thus if the standard proportion ever married z years after first marriage begins is $G_s(z)$, the first marriage cumulative distribution of a cohort can be described as

$$G\ (x) = C.G_s(\frac{x-a_o}{k})$$

where C is the ultimate proportion ever-married in the cohort,

a_o is the age at which first marriage begins,

and k is the number of years equivalent to one year in the standard.

Thus if k = 0.5, marriage occurs twice as fast as in the standard population.

The standard proportions ever married were tabulated at intervals of one tenth of a year (Coale, 1971). An analytical expression for first marriage frequencies was later developed by McNeil (Coale and McNeil, 1972):

$$g(x) = \frac{0.19465}{k} \exp\{- \frac{0.174}{k} (x-a_o-6.06k)$$

$$- \exp [- \frac{0.2881}{k} (x-a_o-6.06k)]\}$$

G(x) can be calculated numerically by integration:

$$G(x) = \int_{a_o}^{x} g(z)\, dz$$

with appropriate estimates of a_o and k. This function describes the pattern of nuptiality of a cohort , and should be multiplied by the appropriate value of C to obtain the correct level of proportions ever married. In their model fertility schedules, Coale and Trussell assume C = 1.0, since only the pattern of fertility is of interest.

Marital fertility

The marital fertility function used in the Coale-Trussell fertility model is a composite of natural fertility, as defined by Henry (1961), and a typical pattern of departure from natural fertility.

Henry defined natural fertility as that which occurs in the absence of voluntary control of births. He defined voluntary control as behaviour which affects fertility in a way which is related to parity. Specifically, control of birth increases as parity increases.

The ratio of a marital fertility schedule, r(x), to natural fertility, n(x) is

$$\frac{r(x)}{n(x)} = M \,.\, e^{m.v(x)} \qquad 3.1$$

The scale factor, M, serves to equate r(x) to n(x) for some chosen value of x. This factor is not important in the model since only the pattern of fertility is of interest. The function v(x) represents a pattern of birth control expressing the extent to which older women in contracepting populations reduce their fertility. The parameter, m, expresses the degree of control: if $m = 0$ there is no control and r(x) and n(x) have the same shape; as m increases r(x) departs from n(x).

The actual values of n(x) and v(x) for $20 \leq x < 50$ were derived empirically, n(x) from ten of Henry's natural fertility populations and v(x) from 43 marital fertility schedules for 1965, using equation 3.1. The values appear in Table 3.1 by 5 year age groups, though single year values (which can be consulted in the computer program as FNAT and DEP in Appendix 3.2) were used to produce the model fertility schedules.

Model fertility

Given the functions of proportions ever married, G(x), and marital fertility, r(x), as described above, the

Table 3.1: Natural fertility and voluntary control schedules by age

Age x	Natural fertility n(x)	Voluntary control v(x)
15-19	.4112	0.000
20-24	.4597	0.000
25-29	.4309	-0.279
30-34	.3946	-0.677
35-39	.3223	-1.042
40-44	.1671	-1.414
45-49	.0237	-1.671

age pattern of fertility of all women can be written as

$$f(x) = G(x) \, . \, r(x)$$
$$= G(x) \, . \, n(x) \, e^{m \, v(x)}$$

The model assumes that there is no extra-marital fertility, coupled with no marital dissolution before the end of the childbearing ages. Coale and Trussell suggest that, where these conditions are not met, an adequate fit to age-specific fertility may be obtained by the use of nuptiality and marital fertility parameters that deviate slightly from the actual population values. (The estimation of a_0, k and m from observed data appears in Appendix 3.1.) Thus, illegitimate births and premarital conceptions at early ages can be taken into account by choosing a slightly smaller a_0 and a slightly larger k than the observed population values. At older ages, illegitimacy can be regarded as a slight increase in marital fertility and can be allowed for by decreasing m slightly. Conversely, marital dissolution can be regarded as reduced marital fertility, attainable by increasing m. The underlying assumption here is that illegitimacy and marital dissolution follow the same sort of age pattern as voluntary birth control.

The set of model fertility schedules produced by Coale and Trussell fall within a region bounded by the limits imposed on the three parameters a_0, k and m.

Age at which first marriage begins is limited to 12.5 to 18 years. The pace of marriage ranges from k = 0.2 (that is 5 times as fast as the Swedish standard) to k = 1.8 (0.56 times as fast). The value of m ranges from 0 (natural fertility) to 3.9 where 1.0 is the average for the 43 schedules involved in determining v(x). Within these bounds, schedules with a mean age of from 24 to 34 years (integral values only) and with a standard deviation from 4.0 to 7.5 years (at half-yearly intervals) were selected for tabulation (Coale and Trussell, 1974). Not all possible mean and standard deviation combinations are attainable within the bounds of the a_0, k and m parameters, and a total of 795 schedules were tabulated.

The Development of the Standard Fertility Schedule

The idea of incorporating a fertility pattern, typical of high fertility populations, into the age scale of the Gompertz model has already been discussed in Chapter 2. The development of such a standard pattern of fertility is the subject of this section.

Determination of area of interest

The standard is based on the Coale-Trussell model of fertility described above. The published set of schedules proved to be too narrow in their range of parameters. In particular, early marrying, high fertility populations are not properly represented, primarily because the lower limit of 12.5 years on the age at which first marriage begins is not low enough. It was therefore necessary to generate these extra fertility schedules. The computer program published by Coale and Trussell (1974) was modified to allow for smaller values of a_o. This and other modifications are described in Appendix 3.2, where the final modified program is reproduced. Because of the way in which the Coale-Trussell set of schedules is presented (by increments in the mean and standard deviation, rather than by equally spaced increments in the generating parameters a_o, k and m), it was found necessary and convenient to generate all the schedules of interest.

High fertility populations are characterised by mean ages of fertility, μ, of about 27 to 29 years, with standard deviations, σ, of more than 6. Accordingly, the area of interest was at first defined as

$$27.0 \le \mu \le 29.0$$
$$6.25 \le \sigma \le 6.75$$

with positive skewness. Schedules were generated from the following parameter values

$10.0 \le a_o \le 15.0$ in steps of 0.5
$.1 \le k \le 1.8$ in steps of 0.1
$0 \le m \le 1.6$ in steps of 0.2

The lower limit of 10 years on a_o was a convenient cut-off point and allowed for maximum flexibility in the level of early fertility. The upper limit of 15 years was based on the patterns of fertility attainable for different values of a_o. Patterns for schedules based on $a_o > 15.0$ were more like those of developed countries (schedules from the United Nations Demographic Yearbook 1969) than of high fertility populations. The limit of 1.8 on k was based on the findings of Lesthaeghe (1971). The limits on m were automatically determined by the other limits.

Not all of the possible combinations were generated since they would clearly lead to μ or σ values outside the stated ranges (the Coale-Trussell published schedules were used as a guide). Some combinations were rejected on calculation of their μ and σ values. Others were rejected on closer scrutinisation of the parameter values for unlikely combinations. For example, within the required range of σ, values of μ of greater than 28.0 years cannot be obtained for $a_o = 10.0$ unless both k and m are greater than one, an unlikely situation of early but slow marriage coupled with a reasonable degree of fertility control. In fact, it was found that values of σ from 6.25 to 6.75 (for $27.0 \le \mu \le 29.0$) could only be attained for high values of m: for $a_0 = 10.0$, m is generally greater than 1.0, and even for $a_0 = 15.0$ m is greater than 0.4. In order to be able to attain smaller values of m within the same range of μ, it was necessary to allow higher values of σ. The area of interest was thus redefined as

$$27.0 \le \mu \le 29.0$$
$$6.75 \le \sigma \le \infty$$
$$\text{for} \quad 10.0 \le a_0 \le 15.0 \quad \text{in steps of } 0.5.$$

The parameters, k and m, were automatically determined as

$$0.1 \le k \le 1.3$$
$$0 \le m \le 1.0$$

and in practice, the upper limit on σ was 8.0. Figures 3.1, 3.2 and 3.3 show the combinations of k and m, for given values of a_0, that result in the required μ and σ values. It can be seen from these graphs that the higher values of k and m occur for low values of a_0 only. Such combinations are unlikely and led to further restrictions on the value of m such that only $0 \le m \le 0.6$ were included. Imposing an upper limit of 21.0 on the singulate mean age at marriage, SMAM (Hajnal, 1953) where

$$\text{SMAM} = a_0 + 11.37\,k,$$

Figure 3.1: Values of k and m resulting in $\mu = (27,29)$, $\sigma > 6.75$ for $a_o = 10.0$.

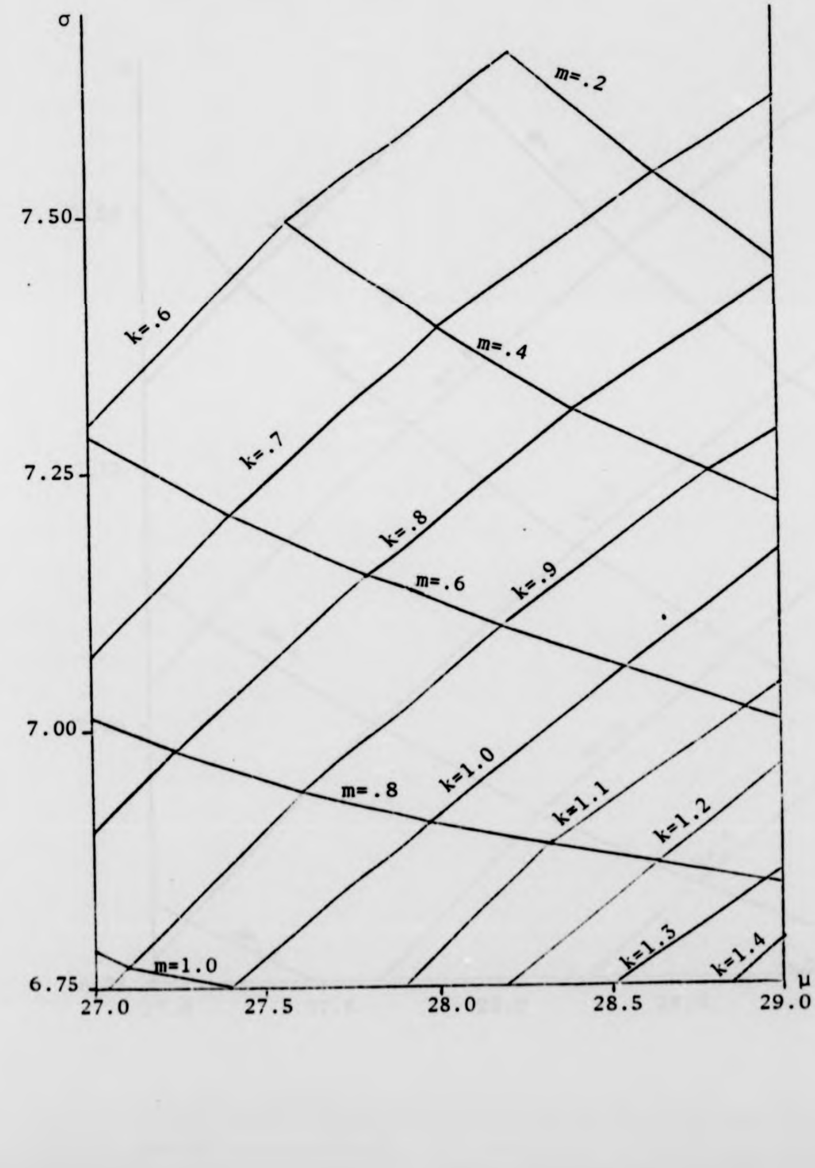


Figure 3.2: Values of k and m resulting in $\mu = (27,29)$, $\sigma > 6.75$ for $a_o = 12.5$

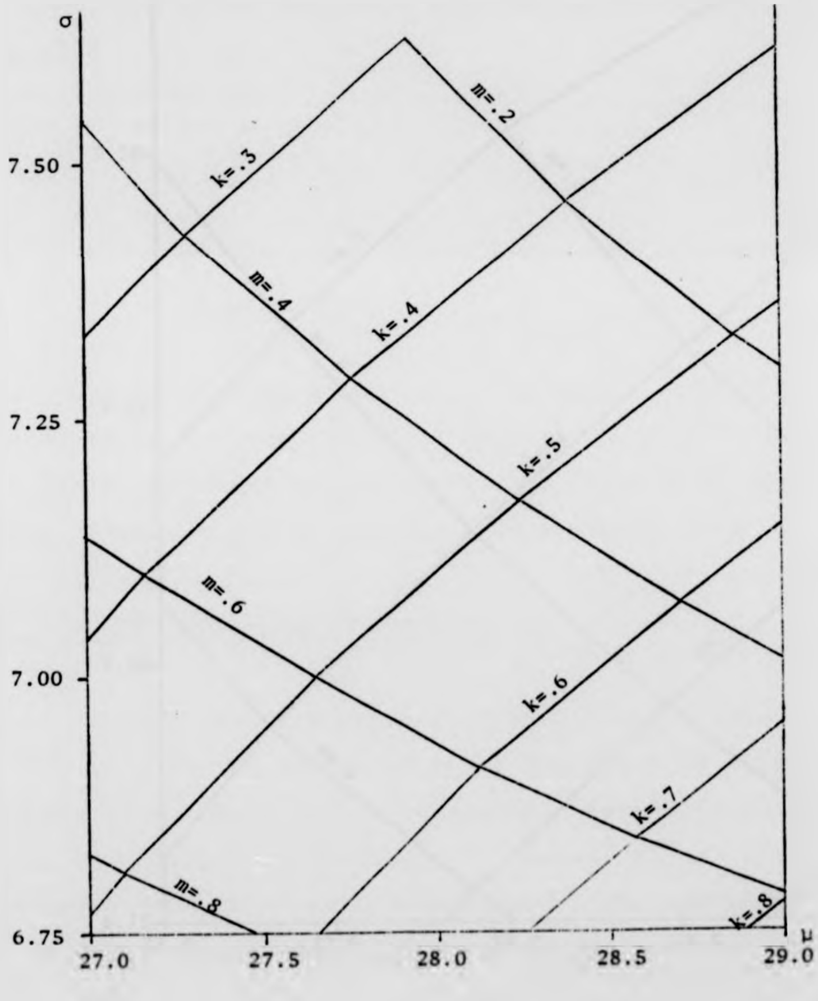


Figure 3.3: Values of k and m resulting in $\mu = (27,29)$, $\sigma > 6.75$ for $a_o = 15.0$

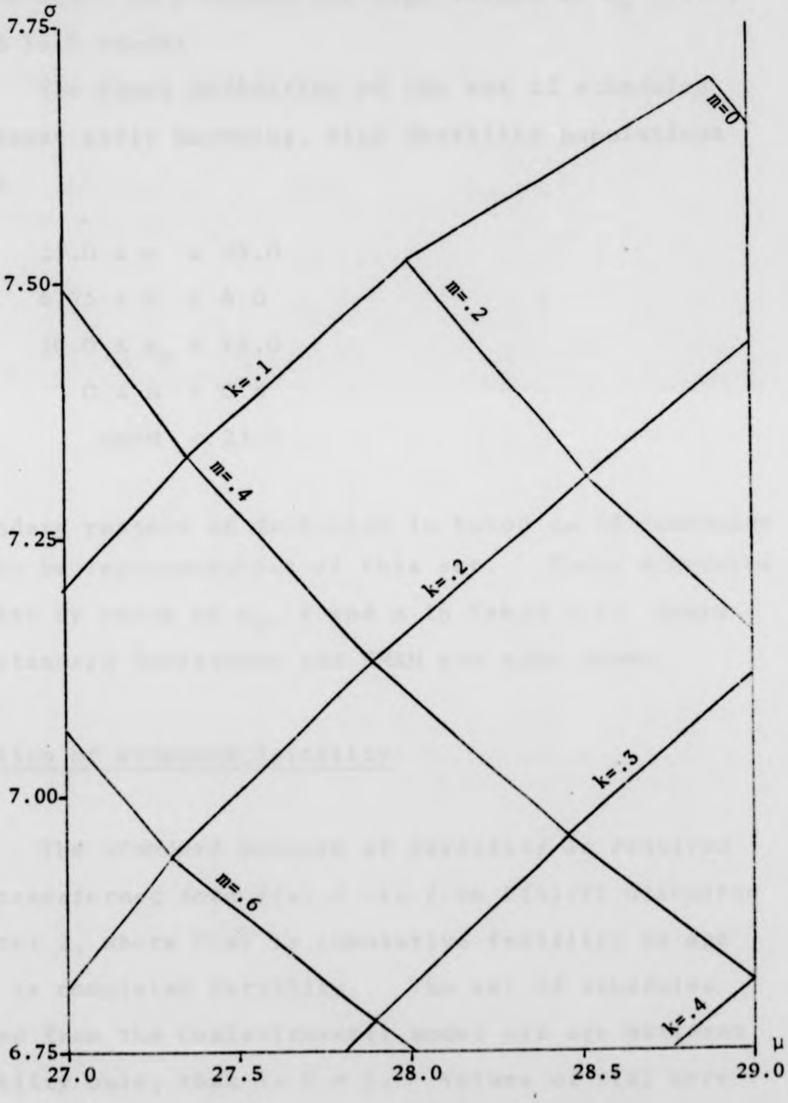
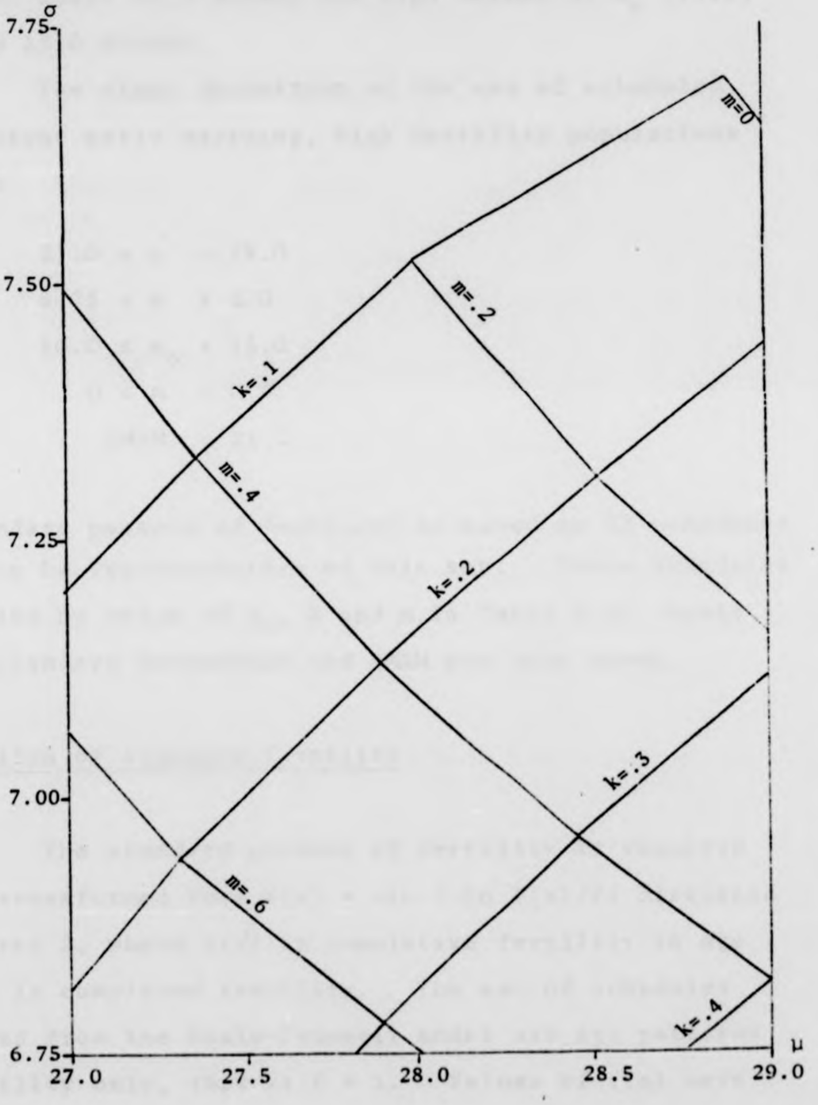


Figure 3.3: Values of k and m resulting in $\mu = (27,29)$, $\sigma > 6.75$ for $a_o = 15.0$



afforded a means of restricting k for given values of a_0, shown in Table 3.2. In fact, this restriction replaced the upper limit on μ except for high values of a_0 (14.0, 14.5 and 15.0 years).

The final definition of the set of schedules to represent early marrying, high fertility populations was thus

$$\begin{aligned} 27.0 \le\ & \mu \le 29.0 \\ 6.75 \le\ & \sigma \le 8.0 \\ 10.0 \le\ & a_0 \le 15.0 \\ 0 \le\ & m \le 0.6 \\ & \text{SMAM} \le 21.0 \end{aligned}$$

The standard pattern of fertility is based on 33 schedules chosen to be representative of this set. These schedules are listed by value of a_0, k and m in Table 3.3; their means, standard deviations and SMAM are also shown.

Calculation of standard fertility

The standard pattern of fertility is required in the transformed form $Y(x) = -\ln(-\ln F(x)/F)$ discussed in Chapter 2, where F(x) is cumulative fertility to age x and F is completed fertility. The set of schedules generated from the Coale-Trussell model are age patterns of fertility only, that is F = 1. Values of Y(x) were calculated for each of the 33 schedules, and differences were obtained:

Table 3.2: Upper limits on k for values of a_o

a_o	Limit on k imposed by		
	SMAM		μ
	Maximum k	SMAM	maximum k
10.0	0.9	20.23	1.3
10.5	0.9	20.73	1.2
11.0	0.8	20.10	1.1
11.5	0.8	20.60	0.9
12.0	0.7	19.96	0.8
12.5	0.7	20.46	0.7
13.0	0.7	20.96	0.7
13.5	0.6	20.32	0.6
*14.0	0.6	20.82	0.5
*14.5	0.5	20.19	0.4
*15.0	0.5	20.69	0.4

* ages at which the upper limit on μ takes effect.

Table 3.3: Generating parameters (a_o, k and m) and derived statistics (μ, σ and SMAM) of schedules used to calculate standard fertility

	Generating parameters			Derived statistics		
No.	a_o	k	m	μ	σ	SMAM
1	10.0	0.7	0.2	28.65	7.54	17.96
2	10.0	0.7	0.6	27.41	7.21	17.96
3	10.0	0.9	0.4	28.79	7.25	20.23
4	10.5	0.6	0.2	28.45	7.56	17.32
5	10.5	0.6	0.6	27.20	7.22	17.32
6	10.5	0.8	0.4	28.63	7.24	19.60
7	11.0	0.5	0.2	28.22	7.59	16.68
8	11.0	0.6	0.6	27.42	7.14	17.82
9	11.0	0.8	0.4	28.85	7.16	20.10
10	11.5	0.4	0.4	27.34	7.46	16.05
11	11.5	0.5	0.6	27.19	7.16	17.18
12	11.5	0.6	0.2	28.87	7.39	18.32
13	12.0	0.4	0.2	28.18	7.56	16.55
14	12.0	0.5	0.6	27.41	7.08	17.68
15	12.0	0.7	0.4	28.91	7.07	19.96
16	12.5	0.3	0.2	27.93	7.62	15.91
17	12.5	0.4	0.6	27.17	7.10	17.05
18	12.5	0.5	0.2	28.86	7.33	18.18
19	13.0	0.2	0.4	27.03	7.51	15.27
20	13.0	0.4	0.2	28.61	7.37	17.55
21	13.0	0.5	0.6	27.90	6.91	18.69
22	13.5	0.2	0.2	27.87	7.61	15.77
23	13.5	0.3	0.6	27.13	7.06	16.91
24	13.5	0.4	0.2	27.84	7.27	18.05
25	14.0	0.2	0.2	28.07	7.51	16.27
26	14.0	0.3	0.6	27.37	6.97	17.41
27	14.0	0.5	0.4	28.99	6.89	19.69
28	14.5	0.1	0.2	27.80	7.62	15.64
29	14.5	0.2	0.6	27.08	7.03	16.77
30	14.5	0.3	0.2	28.81	7.23	17.91
31	15.0	0.1	0.4	27.37	7.33	16.14
32	15.0	0.2	0.2	28.52	7.31	17.27
33	15.0	0.3	0.6	27.88	6.78	18.41

$$\Delta Y(x \text{ to } x+4) = Y(x+5) - Y(x)$$

The age specific fertility rates, cumulative fertility, $Y(x)$ and $\Delta Y(x)$ values for the 33 schedules are shown in Tables 3.4 to 3.7.

Averaging was done over all 33 schedules for the three ΔY values covering ages 25 to 39. These three averages were taken as the standard values, $\Delta Y_s(25\text{-}29)$, $\Delta Y_s(30\text{-}34)$ and $\Delta Y_s(35\text{-}39)$, where the subscript s denotes standard. For the ages outside of this central range, weight was given to those schedules with high fertility at young or old ages, so that the standard pattern of fertility is more representative of distributions with a relatively large proportion of fertility in the tails. The transformed Gompertz model is thus designed to fit better in the tails to distributions where the contribution of fertility at young or old ages is substantial than to distributions with an insignificant proportion of fertility in the tails.

About half of the 33 schedules were averaged to obtain the ΔY_s values at younger and older ages. For young ages, age specific fertility for the first two age groups were added together, and the seventeen schedules with $f(10\text{-}19) > 0.15$ were selected for inclusion. These 17 schedules are marked * in Tables 3.4 to 3.7. For the later childbearing ages, the 16 schedules with $f(35\text{-}49) > 0.21$ (marked + in the Tables 3.4 to 3.7) were used to obtain ΔY values for ages 39-44. The final three

Table 3.4: Age specific fertility rates by 5 year age groups for the 33 schedules used to calculate standard fertility.

SCHEDULE NUMBER	10-14	15-19	AGE 20-24	GROUPS 25-29	30-34	35-39	40-44	45-49
+ 1	.01150	.12640	.21475	.22183	.17248	.14530	.07107	.01009
* 2	.01471	.15133	.23470	.23534	.17574	.11500	.04431	.00627
+ 3	.00742	.10936	.22436	.23792	.20000	.14458	.06031	.00894
+ 4	.01147	.13560	.22426	.21453	.14406	.14724	.05401	.00980
* 5	.01062	.10043	.23001	.23514	.17079	.11194	.04707	.00606
+ 6	.00728	.11430	.23043	.23749	.19847	.14069	.06428	.00807
*+ 7	.01165	.14513	.22277	.21121	.13437	.13432	.06704	.00451
* 8	.00976	.15130	.25002	.23962	.17461	.11457	.04000	.00520
+ 9	.00513	.10562	.22950	.24124	.23140	.14425	.05376	.00889
* 10	.01127	.17272	.24443	.21006	.17004	.11123	.05413	.00727
* 11	.00541	.16244	.26407	.23022	.16445	.11077	.04714	.00539
+12	.00569	.11745	.22444	.22574	.14057	.14417	.07190	.01021
*+13	.00155	.13041	.23175	.21357	.13179	.13596	.06036	.00942
* 14	.00541	.15159	.26605	.23314	.17346	.11344	.04028	.00614
+15	.00175	.09406	.23358	.24462	.20191	.14373	.05534	.00481
* 16	.00312	.15556	.23193	.21156	.17545	.13126	.06436	.00315
* 17	.00533	.15530	.26474	.22556	.16784	.10936	.04561	.00592
+18	.00274	.11604	.23097	.22137	.14458	.14559	.07123	.01011
* 19	.01003	.19743	.24308	.20955	.15383	.11404	.05213	.00700
+20	.00720	.12413	.23537	.22570	.10907	.14248	.06104	.00480
21	.00164	.12430	.26044	.24105	.13201	.11755	.05107	.00647
* 22	.00394	.17422	.23238	.29058	.17540	.13208	.06400	.00903
* 23	.00152	.16440	.21371	.22008	.15609	.10836	.04509	.00506
+24	.00066	.11456	.23831	.22424	.19372	.14563	.07057	.01001
* 25	.00105	.16256	.23515	.21057	.17352	.13443	.06313	.00924
* 26	.00050	.15315	.21907	.23159	.17019	.11164	.04723	.00600
+27	.00017	.09412	.24743	.25015	.20151	.14702	.06455	.00807
+28	.00159	.13420	.23120	.20130	.17559	.13110	.06352	.00401
* 29	.00014	.17544	.21500	.22447	.15436	.10743	.04551	.00500
+30	.00105	.11314	.24343	.22763	.19161	.14430	.06492	.00992
* 31	0.00000	.14496	.25252	.21066	.15553	.11612	.05503	.00720
+32	0.00100	.13134	.24435	.22213	.14577	.13159	.06778	.00462
33	0.00100	.11544	.26572	.24021	.17405	.11722	.04455	.00634

* Schedules used to obtain standard fertility at ages 10-24

\+ Schedules used to obtain standard fertility at ages 40-49

Table 3.5: Cumulative fertility by 5 year age groups for the 33 schedules used to calculate standard fertility.

SCHEDULE	AGE GROUPS							
NUMBER	15	20	25	30	35	40	45	50
+ 1	.01150	.13848	.35822	.58700	.77254	.91584	.98891	1.00000
* 2	.01371	.15504	.41476	.65128	.82492	.96042	.99173	1.00000
+ 3	.00792	.11728	.34164	.57757	.74016	.92874	.99105	1.00000
+ 4	.01147	.14707	.37133	.59005	.77192	.92114	.99820	1.00000
* 5	.01352	.17934	.43400	.66174	.83032	.95627	.99394	1.00000
+ 6	.00728	.12158	.35201	.59051	.74637	.92107	.99133	1.00000
*+ 7	.01165	.13778	.39555	.53176	.74513	.92145	.99049	1.00000
* 8	.00975	.16106	.42114	.65575	.83042	.94000	.99390	1.00000
+ 9	.00513	.10976	.33826	.57450	.78090	.92516	.99112	1.00000
* 10	.01327	.16564	.43249	.64433	.81037	.93369	.99273	1.00000
+ 11	.00941	.11235	.43643	.65565	.83610	.94657	.99401	1.00000
+12	.00563	.12284	.34734	.57413	.76970	.91789	.98979	1.00000
*+13	.00785	.13616	.38441	.50346	.73727	.92622	.99050	1.00000
* 14	.00541	.15750	.42355	.65603	.83214	.94554	.99366	1.00000
+15	.00275	.10151	.33534	.55001	.73192	.92505	.99119	1.00000
* 16	.00412	.17388	.40551	.61507	.77302	.92627	.99054	1.00000
* 17	.00543	.17121	.44050	.67006	.83790	.94741	.99004	1.00000
+18	.00274	.12847	.34426	.57713	.77170	.91364	.98484	1.00000
* 19	.01003	.21747	.45265	.65220	.82603	.94125	.99300	1.00000
+20	.00220	.13033	.30500	.53860	.77508	.92110	.99920	1.00000
21	.00163	.12594	.37443	.60641	.82310	.94765	.99353	1.00000
* 22	.00348	.17820	.41058	.61944	.74484	.92592	.99492	1.00000
* 23	.00192	.17142	.44531	.67350	.83109	.94405	.99414	1.00000
+24	.00085	.11542	.35213	.58051	.77379	.91342	.98444	1.00000
* 25	.00165	.13343	.40010	.61767	.79119	.92562	.99076	1.00000
* 26	.00050	.13348	.43145	.65554	.83573	.94576	.99400	1.00000
+27	.00017	.03500	.33243	.58318	.78478	.92569	.99133	1.00000
+28	.00053	.13373	.41441	.62721	.77637	.92747	.99079	1.00000
* 29	.00013	.17536	.45113	.67710	.84137	.94459	.99620	1.00000
+30	.00005	.11319	.35562	.58625	.77506	.92416	.99004	1.00000
* 31	0.00000	.13434	.43536	.65252	.82184	.93117	.99250	1.00000
+32	0.00000	.13134	.37577	.59175	.79212	.92206	.99135	1.00000
33	0.00000	.11444	.40076	.64191	.82538	.94380	.99356	1.00000

* Schedules used to obtain standard fertility at ages 10-24
\+ Schedules used to obtain standard fertility at ages 40-49

Table 3.6: Y values by 5 year age groups for the 33 schedules used to calculate standard fertility.

SCHEDULE	AGE GROUPS							
NUMBER	15	20	25	30	35	40	45	50
+ 1	-1.49624	-.68135	-.02627	.60766	1.17453	2.46424	4.59081	********
* 2	-1.95015	-.59956	.14143	.55337	1.57262	2.85149	5.00146	********
+ 3	-1.57665	-.70224	-.07136	.69510	1.39330	2.54000	4.71236	********
+ 4	-1.59909	-.63058	.00437	.58621	1.30887	2.59002	4.62057	********
* 5	-1.45782	-.55730	.11492	.84119	1.70452	2.89625	5.10204	********
+ 6	-1.59381	-.74535	-.00316	.53777	1.42577	2.59058	4.74543	********
*+ 7	-1.59457	-.61342	.00596	.57749	1.41920	2.53024	4.55024	********
* 8	-1.53204	-.59212	.14526	.55281	1.60296	2.81218	5.07335	********
+ 9	-1.58244	-.79216	-.08080	.69598	1.39712	2.55376	4.71129	********
* 10	-1.48402	-.52004	.17046	.63477	1.61333	2.75876	4.92017	********
* 11	-1.54032	-.55430	.16737	.90266	1.72030	2.90783	5.11432	********
+ 12	-1.64653	-.74004	-.15573	.53495	1.34034	2.45715	4.57151	********
*+ 13	-1.57632	-.61201	.05543	.58806	1.43050	2.54082	4.60150	********
* 14	-1.53515	-.61028	.15159	.87346	1.64418	2.83324	5.69037	********
+ 15	-1.77447	-.82570	-.00743	.60749	1.40239	2.55125	4.72588	********
* 16	-1.57135	-.55392	.14249	.72444	1.46141	2.50426	4.68108	********
* 17	-1.55523	-.59807	.15586	.91533	1.73245	2.91345	5.12572	********
+ 18	-1.77155	-.75946	-.05054	.59840	1.35133	2.48654	4.58417	********
* 19	-1.52645	-.43245	.23211	.88925	1.05482	2.79757	4.95743	********
+ 20	-1.81128	-.71141	-.10537	.53808	1.33566	2.44058	4.62971	********
21	-1.85898	-.72650	.01222	.50451	1.63638	2.62911	5.03754	********
* 22	-1.70058	-.54515	.11532	.73017	1.47135	2.57554	4.59795	********
* 23	-1.83302	-.58695	.21259	.92525	1.74052	2.73964	5.13975	********
+ 24	-1.95926	-.76573	-.04228	.69126	1.36174	2.47663	4.57315	********
* 25	-1.42435	-.53941	.13770	.71350	1.45151	2.55922	4.61725	********
* 26	-2.02723	-.62678	.17502	.84055	1.71784	2.90508	5.11236	********
+ 27	-2.15744	-.40224	-.09632	.61755	1.41737	2.57628	4.74300	********
+ 28	-2.02065	-.52625	.12637	.74577	1.47478	2.54413	4.70444	********
* 29	-2.13461	-.55141	.22190	.94141	1.75003	2.94167	5.14742	********
+ 30	-2.24982	-.77173	-.11061	.62797	1.37126	2.49964	4.60306	********
* 31	********	-.52524	.10657	.65117	1.52454	2.76844	4.92293	********
+ 32	********	-.79165	.02145	.55177	1.41675	2.51482	4.63427	********
33	********	-.77044	.06431	.83122	1.53831	2.85003	5.05102	********

* Schedules used to obtain standard fertility at ages 10-24
+ Schedules used to obtain standard fertility at ages 40-49

Table 3.7: Values of ΔY by 5 year age groups for the 33 schedules used to calculate standard fertility.

SCHEDULE NUMBER	AGE 10-14	15-19	20-24	GROUPS 25-29	30-34	35-39	40-44	45-49
+ 1	1.10347	.81462	.65534	.53191	.74580	1.11475	2.12152	*********
* 2	1.13046	.86751	.73004	.71296	.91712	1.13[illegible]	2.20371	*********
+ 3	1.13497	.81415	.69691	.67148	.73720	1.15970	2.10635	*********
+ 4	1.21173	.84622	.60005	.63272	.74479	1.11413	2.12065	*********
* 5	1.24501	.90072	.73011	.71216	.81733	1.18714	2.20559	*********
+ 6	1.13415	.84846	.70221	.69191	.78800	1.15481	2.10405	*********
*+ 7	*********	.88022	.65130	.62943	.74171	1.11104	2.12000	*********
* 8	*********	.93032	.74737	.71756	.82015	1.18922	2.20714	*********
+ 9	*********	.85958	.71216	.69548	.79124	1.15854	2.10552	*********
* 10	*********	.94376	.[illegible]	.65331	.77356	1.14545	2.10141	*********
* 11	*********	.97603	.75167	.71529	.91764	1.18753	2.20547	*********
+ 12	*********	.90944	.[illegible]41	.59965	.75138	1.11[illegible]2	2.12230	*********
*+ 13	*********	.96532	.67133	.62276	.74082	1.11131	2.11265	*********
* 14	*********	1.02039	.76617	.72151	.82072	1.13[illegible]11	2.2070[illegible]	*********
+ 15	*********	.94927	.73777	.59592	.73490	1.15[illegible]16	2.16532	*********
* 16	*********	1.01142	.66239	.62[illegible]	.73857	1.10795	2.11372	*********
* 17	*********	1.05715	.76671	.71[illegible]59	.81712	1.13700	2.20527	*********
+ 18	*********	1.01535	.70535	.64[illegible]04	.75193	1.11660	2.12223	*********
* 19	*********	1.07362	.68496	.65915	.76856	1.14275	2.16336	*********
+ 20	*********	1.04947	.70544	.64345	.74758	1.11[illegible]92	2.12112	*********
21	*********	1.13048	.80072	.73024	.82787	1.14273	2.20[illegible]42	*********
* 22	*********	1.16443	.66148	.91[illegible]04	.73518	1.10714	2.11842	*********
* 23	*********	1.26734	.77863	.71590	.81598	1.18632	2.20101	*********
+ 24	*********	1.18453	.72705	.65156	.75151	1.11[illegible]	2.12195	*********
* 25	*********	1.33154	.68011	.62579	.73402	1.10971	2.11303	*********
* 26	*********	1.40045	.60180	.72353	.81328	1.18794	2.20550	*********
+ 27	*********	1.25570	.80572	.71407	.79983	1.15741	2.10082	*********
+ 28	*********	1.49321	.[illegible]552	.51[illegible]	.79000	1.10555	2.11110	*********
* 29	*********	1.63734	.76390	.71352	.91452	1.19563	2.20575	*********
+ 30	*********	1.51429	.74812	.55159	.75029	1.11517	2.12161	*********
* 31	*********	*********	.71386	.65280	.77248	1.14479	2.10115	*********
+ 32	*********	*********	.72430	.66130	.74476	1.11221	2.12045	*********
33	*********	*********	.86081	.74189	.82710	1.19[illegible]72	2.20749	*********

* Schedules used to obtain standard fertility at ages 10-24

\+ Schedules used to obtain standard fertility at ages 40-49

age groups were included to allow weighting to be based on at least 10 per cent of fertility: the last two age groups contain only 5-8 per cent of fertility. These values of average ΔY values appear in Table 3.8. Since $Y(10) = -\infty$ and $Y(50) = \infty$, ΔY values for the first and last age groups are also infinite. Because the average ΔY values for early and late ages are based on only half of the schedules, adjustment factors are needed to bring them to the same level as values for the middle child-bearing ages. These factors were calculated as

$$k_1 = \frac{\text{average } \Delta Y(25\text{-}39) \text{ for all 33 schedules}}{\text{average } \Delta Y(25\text{-}39) \text{ for the 17 high early fertility schedules}}$$

$$k_2 = \frac{\text{average } \Delta Y(25\text{-}39) \text{ for all 33 schedules}}{\text{average } \Delta Y(25\text{-}39) \text{ for the 16 high late fertility schedules}}$$

where $\Delta Y(25\text{-}39) = \Delta Y(25\text{-}29) + \Delta Y(30\text{-}34) + \Delta Y(35\text{-}39)$.

$$\text{Thus } k_1 = \frac{0.67436 + 0.77872 + 1.14730}{0.67976 + 0.78638 + 1.15692}$$

$$= \frac{2.60038}{2.62306} = 0.99135$$

is used to adjust the average $\Delta Y(15\text{-}19)$ and $\Delta Y(20\text{-}24)$ values to give $\Delta Y_s(15\text{-}19)$ and $\Delta Y_s(20\text{-}24)$. Similarly

$$k_2 = \frac{0.67436 + 0.77872 + 1.14730}{0.65492 + 0.76042 + 1.12689} = \frac{2.60038}{2.54223} = 1.02287$$

is used to adjust average $\Delta Y(40\text{-}44)$ to give $\Delta Y_s(40\text{-}44)$. The adjusted averages, ΔY_s, are shown in Table 3.8.

age groups were included to allow weighting to be based on at least 10 per cent of fertility: the last two age groups contain only 5-8 per cent of fertility. These values of average ΔY values appear in Table 3.8. Since $Y(10) = -\infty$ and $Y(50) = \infty$, ΔY values for the first and last age groups are also infinite. Because the average ΔY values for early and late ages are based on only half of the schedules, adjustment factors are needed to bring them to the same level as values for the middle child-bearing ages. These factors were calculated as

$$k_1 = \frac{\text{average } \Delta Y(25\text{-}39) \text{ for all 33 schedules}}{\text{average } \Delta Y(25\text{-}39) \text{ for the 17 high early fertility schedules}}$$

$$k_2 = \frac{\text{average } \Delta Y(25\text{-}39) \text{ for all 33 schedules}}{\text{average } \Delta Y(25\text{-}39) \text{ for the 16 high late fertility schedules}}$$

where $\Delta Y(25\text{-}39) = \Delta Y(25\text{-}29) + \Delta Y(30\text{-}34) + \Delta Y(35\text{-}39)$.

Thus $k_1 = \dfrac{0.67436 + 0.77872 + 1.14730}{0.67976 + 0.78638 + 1.15692}$

$$= \frac{2.60038}{2.62306} = 0.99135$$

is used to adjust the average $\Delta Y(15\text{-}19)$ and $\Delta Y(20\text{-}24)$ values to give $\Delta Y_s(15\text{-}19)$ and $\Delta Y_s(20\text{-}24)$. Similarly

$$k_2 = \frac{0.67436 + 0.77872 + 1.14730}{0.65492 + 0.76042 + 1.12689} = \frac{2.60038}{2.54223} = 1.02287$$

is used to adjust average $\Delta Y(40\text{-}44)$ to give $\Delta Y_s(40\text{-}44)$. The adjusted averages, ΔY_s, are shown in Table 3.8.

Table 3.8: Average and standard values of ΔY and standard Y values by 5 year age groups

Age	Average ΔY (1)	k_i (2)	Standard ΔY_s (3 = 1x2)	Exact age x	Standard $Y_s(x)$
				10	$-\infty$
10-14	∞	-	∞		
				15	-1.77306
15-19	1.09120	0.99135	1.08176		
				20	-0.69130
20-24	0.72320	0.99135	0.71694		
				25	0.02564
25-29	0.67436	1.0	0.67436		
				30	0.70000
30-34	0.77872	1.0	0.77872		
				35	1.47872
35-39	1.14730	1.0	1.14730		
				40	2.62602
40-44	2.13486	1.02287	2.18368		
				45	4.80970
45-49	∞	-	∞		
				50	∞

The translation of the ΔY_s values into $Y_s(x)$ values requires the determination of a fixed point. $Y_s(30) = 0.7$ was chosen as this point as a rough average of the Y(30) values for all 33 schedules. The $Y_s(x)$ schedule was thus calculated from this point in the following way:

$$Y_s(15) = Y_s(30) - \Delta Y_s(25\text{-}29) - \Delta Y_s(20\text{-}24) - \Delta Y_s(15\text{-}19)$$

$$Y_s(20) = Y_s(30) - \Delta Y_s(25\text{-}29) - \Delta Y_s(20\text{-}24)$$

$$Y_s(25) = Y_s(30) - \Delta Y_s(25\text{-}29)$$

$$Y_s(30) = Y_s(30)$$

$$Y_s(35) = Y_s(30) + \Delta Y_s(30\text{-}34)$$

$$Y_s(40) = Y_s(30) + \Delta Y_s(30\text{-}34) + \Delta Y_s(35\text{-}39)$$

$$Y_s(45) = Y_s(30) + \Delta Y_s(30\text{-}34) + \Delta Y_s(35\text{-}39) + \Delta Y_s(40\text{-}44)$$

The resulting $Y_s(x)$ values are given in Table 3.8. The rather arbitrary choice of 0.7 as $Y_s(30)$ determines the origin of the standard, x_{os}, defined as the age at which $Y_s(x_{os}) = 0$. The effect of choosing some other value for $Y_s(30)$ would be equivalent to a change in the origin, the implications of which are discussed fully in Chapter 2 and Appendix 2.4.

The single year standard

The development of standard fertility as described above resulted in a schedule by five year age groups, the values of which are applicable to the endpoints of the age groups, that is exact ages 15, 20, 25, ..., 50. Use of this standard set of values in the transformed Gompertz model is therefore restricted to data cumulated to exact ages 15, 20, etc. In order to make the model more general, and in particular to allow for the use of birth history data, standard values at intervals other than these endpoints are needed.

One way of obtaining intermediate values of standard fertility is to fit a mathematical function to the schedule: fertility at any age can then be calculated simply by putting that age into the function equation. This method, however, would be likely to give a better fit to the middle of the distribution than to the tails where the focus of interest in fitting exists. In addition, a mathematical function would not fit exactly to all endpoints (as calculated above) and adjustments would be necessary to maintain a parallel.

A more favourable method of obtaining intermediate values of standard fertility is to use the 33 schedules on which the standard is based. This was the method employed. Single year fertility and ΔY values were calculated. Averaging of the ΔY values was as before: all 33 schedules were averaged for ages 25 to 39; the 17 schedules with high early fertility were averaged for ages 10 to 24; and the 16 schedules with high late fertility for ages 40 to 49. Where ΔY values do not exist, that is where fertility is zero for consecutive ages, averaging was done over those schedules for which ΔY values exist. This led to discrepancies, however, in the sums of the 5 single-year values and the five-year values already calculated, so that slight adjustments had to be made to the single-year values to make them consistent with the five-year values. Rounding errors were similarly dealt with. Single year values of age specific and cumulative fertility and of Y and ΔY appear in Tables 3.9 to 3.12. The adjusted average ΔY values are shown in Table 3.13. The same adjustment factors, k_1 and k_2, were used to bring the early and late fertility ΔY averages into line with those for the middle years. The resulting ΔY_s values are shown in Table 3.13 along with single year $Y_s(x)$ values.

The midpoint standard

To obtain the standard values required in the application of the model to birth history data, average values of cumulative fertility for each five year age group were calculated by

Table 3.9: Age specific fertility by single years for the 33 schedules used to calculate standard fertility.

SCHEDULE			AGE					AGE					AGE		
NUMBER	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	[illegible]	[illegible]	.00113	[illegible]	[illegible]	.01217	.01415	.02574	.03267	.03708	.04105	.04325	.04433	.04531	.04559
*2	[illegible]	[illegible]	.00135	[illegible]	[illegible]	.01453	.02225	.03064	.03099	.04491	.04595	.05093	.05100	.05169	.05116
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00704	.01474	.02158	.02694	.03505	.03950	.04327	.04571	.04735	.04023
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01133	.02043	.02773	.03492	.03478	.04[illegible]72	.04452	.04543	.04581	.04577
*5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01559	.02491	.03291	.04133	.04709	.05652	.05221	.05264	.05224	.03115
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00917	.01532	.02265	.03043	.03573	.04147	.04478	.04897	.04831	.04090
*7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01903	.02232	.03029	.03730	.04181	.04435	.04503	.04507	.04505	.04507
*8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01317	.02137	.03977	.03970	.04408	.05017	.05221	.05299	.05262	.05169
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00747	.01325	.02047	.02838	.03506	.04027	.04404	.04605	.04834	.04920
*10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01744	.02734	.03411	.04353	.04775	.04572	.05018	.04402	.04898	.04779
*11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01433	.02372	.03155	.04205	.04168	.05216	.05354	.05370	.05301	.05167
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00944	.01379	.02367	.03157	.03747	.04149	.04410	.04503	.04647	.04676
*13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01370	.02209	.03149	.03895	.04348	.04576	.04671	.04603	.04652	.04392
*14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01140	.02070	.03101	.04092	.04706	.05182	.05371	.05421	.05375	.05255
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00569	.01146	.01410	.02772	.03514	.04092	.04504	.04701	.04951	.05029
*16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01031	.02645	.03521	.04204	.04552	.04966	.04709	.04506	.04502	.04518
*17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01251	.02343	.03463	.04452	.05071	.05395	.05490	.05470	.05385	.05201
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00710	.01415	.02363	.03225	.03374	.04296	.04533	.04606	.04748	.04753
*19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.02207	.03493	.04226	.04605	.05045	.05093	.05034	.04327	.04800	.04655
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00754	.01044	.02051	.03575	.04188	.04539	.04716	.04780	.04742	.04741
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01315	.02450	.03371	.04143	.05032	.05355	.05502	.05570	.05440
*22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.02313	.03108	.04431	.04994	.04750	.04736	.04507	.04589	.04491
*23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00992	.02311	.03943	.04719	.05325	.05593	.05520	.05546	.05399	.05206
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01472	.01279	.02311	.03335	.04051	.04455	.04720	.04621	.04846	.04019
*25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01327	.02944	.03741	.04400	.04730	.04925	.04512	.04740	.04644	.04570
*26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01534	.01779	.03245	.04504	.05756	.05017	.05703	.05499	.05514	.05325
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00170	.00647	.01506	.02391	.03509	.04311	.04603	.05097	.05247	.05259
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.01850	.03324	.04122	.04570	.04741	.04750	.04716	.04538	.04552	.04459
*29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.02272	.03463	.05093	.05502	.05758	.05705	.05564	.05349	.05177
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00195	.01011	.02291	.03514	.04303	.04717	.04900	.04943	.04923	.04059
*31	[illegible]	[illegible]	[illegible]	[illegible]	.00000	.00629	.03040	.04435	.05043	.05243	.05263	.05188	.05072	.04939	.04709
32	[illegible]	[illegible]	[illegible]	[illegible]	.00000	.00131	.01234	.02906	.04140	.04728	.04939	.04973	.04925	.04848	.04753
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00063	.00074	.02107	.03744	.04910	.05559	.05814	.05850	.05760	.05566

* Schedules used to obtain standard fertility at ages 10-24

Table 3.9 cont'd.

[illegible]	25	26	27	28	29	30	31
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.04140	[illegible]

AGE					AGE		
32	33	34	35	36	37	38	39
03454	.03704	.03328	.03345	.03147	.02443	.02726	.02404
03504	.03257	.03017	.02752	.02545	.02314	.02084	.01636
[illegible]	.03815	.03399	.03377	.03141	.02904	.02658	.02378
03744	.03812	.03338	.03257	.03062	.02802	.02649	.02399
[illegible]	.03153	.02927	.02597	.02455	.02240	.02017	.01778
03483	.03754	.03518	.03293	.03059	.02825	.02543	.02309
03474	.03515	.03384	.03166	.02977	.02702	.02575	.02331
03468	.03258	.02395	.02760	.02523	.02293	.02044	.01818
[illegible]	.03824	.03502	.03375	.03157	.02897	.02643	.02305
[illegible]	.03203	.03002	.02802	.02596	.02392	.02183	.01944
[illegible]	.03135	.02899	.02570	.02440	.02217	.01945	.01756
[illegible]	.03759	.03578	.03391	.03188	.02901	.02780	.02499
03442	.03456	.03314	.03137	.02940	.02754	.02549	.02307
03440	.03210	.02769	.02754	.02494	.02270	.02043	.01799
[illegible]	.03020	.03598	.03368	.03127	.02880	.02637	.02356
03564	.03332	.03225	.03053	.02658	.02480	.02440	.02245
[illegible]	.03103	.02469	.02641	.02413	.02192	.01973	.01737
[illegible]	.03755	.03553	.03355	.03102	.02454	.02736	.02477
03275	.03085	.02642	.02699	.02501	.02304	.02103	.01877
03784	.03625	.03447	.03264	.03067	.02865	.02652	.02401
[illegible]	.03381	.03128	.02581	.02633	.02343	.02153	.01996
03613	.03352	.03198	.03026	.02663	.02556	.02459	.02225
03312	.03070	.02438	.02912	.02307	.02108	.01951	.01719
03471	.03700	.03523	.03336	.03134	.02429	.02710	.02454
03574	.03422	.03753	.03088	.02643	.02703	.02502	.02285
03468	.03136	.02448	.02077	.02440	.02222	.01949	.01780
[illegible]	.03605	.03571	.03336	.03092	.02850	.02601	.02323
[illegible]	.03337	.03172	.03003	.02622	.02436	.02440	.02209
03277	.03030	.02408	.02385	.02362	.02146	.01931	.01700
03446	.03573	.03492	.03306	.03106	.02402	.02686	.02431
03449	.03173	.02474	.02777	.02572	.02370	.02143	.01931
03721	.03561	.03385	.03205	.03011	.02813	.02683	.02357
03543	.03321	.03070	.02626	.02582	.02346	.02111	.01658

Table 3.9 cont'd.

SCHEDULE NUMBER	AGE 25	26	27	28	29	AGE 30	31	32	33	34	AGE 35	36	37	38	39
1	[illegible]	[illegible]	[illegible]	.04473	.04274	.04151	.04005	.03454	.03704	.03528	.03345	.03147	.02943	.02726	.02409
2	[illegible]	[illegible]	[illegible]	.04445	.04273	.04019	.03755	.03505	.03257	.03017	.02752	.02545	.02314	.02084	.01836
3	[illegible]	[illegible]	.04742	.04701	.04575	.04409	.04215	.04020	.03815	.03599	.03377	.03141	.02904	.02658	.02378
4	[illegible]	[illegible]	[illegible]	.04309	.04179	.04069	.03919	.03764	.03812	.03438	.03257	.03062	.02802	.02649	.02399
5	[illegible]	[illegible]	[illegible]	.04410	.04174	.03922	.03658	.03408	.03153	.02927	.02691	.02455	.02240	.02017	.01776
6	.04603	[illegible]	[illegible]	.04009	.04527	.04343	.04145	.03943	.03734	.03516	.03293	.03059	.02825	.02543	.02309
7	.04514	[illegible]	[illegible]	.04227	.04110	.03876	.03824	.03674	.03518	.03346	.03158	.02977	.02702	.02575	.02331
8	.05007	[illegible]	[illegible]	.04502	.04270	.04009	.03741	.03486	.03236	.02995	.02760	.02523	.02293	.02064	.01818
9	[illegible]	.04437	[illegible]	.04755	.04518	.04441	.04239	.04035	.03824	.03502	.03375	.03137	.02897	.02649	.02309
10	[illegible]	.04512	.04304	.04178	.04006	.03803	.03577	.03349	.03202	.03002	.02802	.02596	.02392	.02183	.01949
11	.05012	[illegible]	.04515	.04183	.04150	.03493	.03032	.03380	.03135	.02899	.02570	.02440	.02217	.01945	.01736
12	.04671	[illegible]	.04553	.04453	.04370	.04220	.04013	.03921	.03759	.03578	.03391	.03188	.02901	.02760	.02499
13	[illegible]	[illegible]	.04315	.04205	[illegible]	.03486	.03792	.03842	.03485	.03314	.03137	.02948	.02754	.02549	.02307
14	.05101	.04545	.04715	.04497	.04230	.03869	.03711	.03460	.03210	.02969	.02734	.02496	.02270	.02043	.01799
15	[illegible]	.05023	[illegible]	.04194	.04057	.04406	.04255	.04044	.03820	.03598	.03358	.03127	.02886	.02637	.02356
16	.04430	.04352	[illegible]	.04100	.03978	.03442	.03641	.03544	.03372	.03225	.03053	.02858	.02600	.02440	.02245
17	[illegible]	.04030	[illegible]	.04173	.04130	.03806	.03544	.03347	.03103	.02864	.02641	.02413	.02192	.01973	.01737
18	[illegible]	[illegible]	[illegible]	.04410	.04133	.04214	.04055	.03844	.03735	.03553	.03355	.03102	.02936	.02736	.02477
19	[illegible]	[illegible]	.04138	.04032	.03957	.03866	.03475	.03275	.03085	.02892	.02699	.02501	.02304	.02103	.01877
20	.04514	.04574	.04643	.04309	.04245	.04103	.03944	.03784	.03926	.03441	.03264	.03067	.02865	.02652	.02401
21	.04317	.05157	.04441	.04720	.04472	.04146	.03912	.03644	.03351	.03128	.02681	.02633	.02393	.02153	.01996
22	[illegible]	.04249	.04110	.04005	.03944	.03404	.03059	.03513	.03352	.03196	.03026	.02643	.02556	.02458	.02225
23	.05019	.04304	.04505	.04439	.04042	[illegible]	.03572	.03312	.03070	.02938	.02912	.02367	.02108	.01951	.01718
24	[illegible]	.04505	[illegible]	.04451	.04335	.04192	.04030	.03871	.03706	.03523	.03336	.03134	.02929	.02710	.02454
25	.04677	.04374	[illegible]	.04137	.04014	.03977	.03724	.03576	.03422	.03253	.03080	.02893	.02703	.02502	.02205
26	.05124	.04703	.04076	.04442	.04192	[illegible]	.03549	.03394	.03146	.02908	.02677	.02446	.02222	.01999	.01760
27	.05259	.05169	.05042	.04485	.04705	.04492	.04290	.04034	.03605	.03571	.03336	.03092	.02850	.02601	.02323
28	.04356	.04260	.04149	.04035	.03915	.03780	.03532	.03367	.03337	.03172	.03003	.02822	.02636	.02440	.02209
29	.04471	.04751	.04292	.04293	.04050	.03754	.03525	.03277	.03036	.02808	.02595	.02362	.02146	.01931	.01700
30	.04779	.04081	.04559	.04437	.04307	.04160	.03977	.03836	.03573	.03492	.03306	.03106	.02902	.02686	.02431
31	.04643	.04491	.04315	.04147	.03958	.03771	.03565	.03369	.03173	.02974	.02777	.02572	.02370	.02163	.01931
32	.04657	.04551	.04427	.04405	.04177	.04034	.03875	.03721	.03561	.03385	.03205	.03011	.02813	.02603	.02357
33	.05346	.05184	.04432	.04567	.04024	.04140	.03832	.03583	.03321	.03070	.02626	.02382	.02346	.02111	.01858

Table 3.9 cont'd.

SCHEDULE NUMBER	AGE 40	41	42	43	44	AGE 45	46	47	48	49
+ 1	[illegible]	[illegible]	[illegible]	.01052	.00715	[illegible]	[illegible]	.00168	.00058	.00024
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00174	.00103	.00053	.00014
+ 3	[illegible]	[illegible]	.01310	[illegible]	[illegible]	[illegible]	.00219	.00143	.00077	.00021
+ 4	[illegible]	[illegible]	[illegible]	.01021	[illegible]	.00434	.00274	.00163	.00085	.00023
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00273	[illegible]	.00094	.00051	.00014
+ 6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00367	.00241	.00143	.00074	.00020
+ 7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00422	.00265	.00159	.00063	.00023
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00173	.00102	.00053	.00014
+ 9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00147	.00076	.00021
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00203	.00120	.00062	.00017
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00098	.00051	.00014
+12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00453	[illegible]	.00170	.00059	.00024
+13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00413	[illegible]	.00157	.00052	.00022
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00171	.00101	.00052	.00014
+15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00146	.00076	.00021
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00405	[illegible]	.00153	.00080	.00022
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00267	[illegible]	.00097	.00050	.00014
+18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00164	.00055	.00024
19	[illegible]	[illegible]	[illegible]	.00755	[illegible]	.00313	[illegible]	.00115	.00060	.00016
+20	[illegible]	[illegible]	[illegible]	.01021	[illegible]	.00434	.00274	.00163	.00085	.00023
21	[illegible]	[illegible]	[illegible]	.00721	[illegible]	[illegible]	[illegible]	.00105	.00055	.00015
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00403	[illegible]	.00151	.00079	.00021
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00095	.00050	.00013
+24	[illegible]	[illegible]	[illegible]	[illegible]	.00711	.00444	.00270	.00167	.00087	.00024
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00410	.00255	.00154	.00080	.00022
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00270	[illegible]	.00099	.00051	.00014
+27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00357	.00242	.00143	.00074	.00020
+28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00400	.00252	.00150	.00078	.00021
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.00161	.00095	.00049	.00013
+30	[illegible]	[illegible]	[illegible]	.01034	.00706	[illegible]	.00277	.00165	.00086	.00023
31	[illegible]	[illegible]	[illegible]	.00777	[illegible]	[illegible]	.00211	.00119	.00062	.00017
+32	[illegible]	[illegible]	[illegible]	.01003	[illegible]	[illegible]	[illegible]	.00160	.00084	.00023
33	[illegible]	.01263	.00977	.00797	[illegible]	[illegible]	.00175	.00104	.00054	.00015

+ Schedules used to obtain standard fertility at ages 40-49

Table 3.10: Cumulative fertility by single years[1] for the 33 schedules used to calculate standard fertility.

SCHEDULE NUMBER			AGE					AGE					AGE		
	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	.00000	.00020	.00134	.00537	.01150	.02364	.04237	.06411	.10070	.13445	.17751	.22275	.25732	.31263	.35822
* 2	.00000	.00020	.00154	.00544	.01371	.02424	.05050	.09114	.12013	.16304	.21359	.26482	.31608	.36857	.41974
3	.00000	.00016	.00040	.00307	.00742	.01556	.03171	.05329	.08223	.11726	.15705	.20035	.24600	.29341	.34104
4	.00000	.00012	.00105	.00621	.01147	.02561	.04446	.07257	.10739	.14707	.18059	.23431	.27774	.32556	.37133
* 5	.00000	.00016	.00125	.00499	.01302	.02420	.05322	.08612	.12765	.17456	.22510	.27737	.33001	.38275	.43340
6	.00000	.00008	.00060	.00259	.00728	.01565	.03178	.05443	.09485	.12150	.16305	.20763	.25480	.30311	.35201
* 7	0.00000	.00005	.00074	.00395	.01165	.02615	.04647	.07867	.11597	.15778	.20213	.24776	.29303	.33948	.38555
* 8	0.00000	.00004	.00069	.00412	.00976	.02391	.04430	.07527	.11498	.16196	.21123	.26344	.31643	.36924	.42114
9	0.00000	.00003	.00034	.00152	.00513	.01260	.02545	.04532	.07470	.10476	.15003	.19407	.24073	.28900	.33926
* 10	0.00000	.00001	.00055	.00363	.01327	.03125	.05880	.09071	.13024	.18599	.23570	.28589	.33570	.38468	.43248
* 11	0.00000	.00001	.00037	.00258	.00841	.02374	.04747	.08103	.12366	.17235	.22452	.27505	.33175	.38470	.43543
12	0.00000	.00001	.00022	.00148	.00549	.01444	.03023	.05390	.08547	.12294	.16443	.20853	.25416	.30053	.34739
* 13	0.00000	0.00000	.00014	.00172	.00785	.02155	.04475	.07570	.11409	.15416	.20392	.25003	.29740	.34349	.38991
* 14	0.00000	0.00000	.00011	.00125	.00601	.01731	.03801	.06402	.10985	.15750	.20932	.26303	.31725	.37100	.42355
15	0.00000	0.00000	.00006	.00050	.00275	.00141	.01935	.03495	.06657	.10141	.14274	.18773	.23559	.28510	.33539
* 16	0.00000	0.00000	.00003	.00124	.00412	.02445	.05041	.08512	.12616	.17364	.22054	.26763	.31431	.36033	.40551
* 17	0.00000	0.00000	.00002	.00080	.00513	.01791	.04134	.07598	.12050	.17121	.22519	.28014	.33465	.38849	.44050
18	0.00000	0.00000	.00001	.00042	.00279	.00438	.02446	.04787	.08013	.11487	.16153	.20739	.25424	.30173	.34920
* 19	0.00000	0.00000	.00010	.00091	.01603	.03266	.05657	.10845	.15701	.20747	.25540	.30513	.35000	.40500	.45235
20	0.00000	0.00000	0.00000	.00018	.00520	.00974	.02616	.05267	.09045	.13033	.17572	.22230	.27009	.31850	.36590
21	0.00000	0.00000	0.00000	.00015	.00163	.00743	.02127	.06574	.09150	.12594	.17626	.22942	.28483	.34003	.39443
* 22	0.00000	0.00000	0.00000	.00004	.00378	.02106	.04818	.08695	.13126	.17420	.22380	.27316	.31903	.36567	.41050
* 23	0.00000	0.00000	0.00000	.00005	.00142	.01184	.03445	.07139	.11059	.17102	.22775	.28402	.33948	.39347	.44553
24	0.00000	0.00000	0.00000	.00002	.00135	.00558	.01834	.04155	.07491	.11542	.16026	.20746	.25567	.30414	.35233
* 25	0.00000	0.00000	0.00000	0.00000	.00105	.01133	.03021	.07262	.11063	.16333	.21217	.26029	.30770	.35440	.40010
* 26	0.00000	0.00000	0.00000	0.00000	.00040	.00504	.02312	.05527	.10132	.15388	.21005	.26708	.32357	.37871	.43175
27	0.00000	0.00000	0.00000	0.00000	.00017	.00187	.00835	.02340	.04931	.08500	.12611	.17614	.22712	.27958	.33243
28	0.00000	0.00000	0.00000	0.00000	.00023	.01003	.04932	.09054	.13032	.18373	.23133	.27849	.32407	.37039	.41498
* 29	0.00000	0.00000	0.00000	0.00000	.00013	.00576	.02045	.06611	.11903	.17506	.23264	.28909	.34537	.39926	.45103
30	0.00000	0.00000	0.00000	0.00000	.00005	.00201	.01211	.03502	.07016	.11319	.16036	.20935	.25800	.30803	.35562
* 31	0.00000	0.00000	0.00000	0.00000	.00000	.00624	.03713	.08148	.13191	.18434	.23098	.28956	.33958	.38847	.43666
32	0.00000	0.00000	0.00000	0.00000	.00000	.00131	.01364	.04270	.08910	.13139	.16077	.23031	.27976	.32824	.37577
33	0.00000	0.00000	0.00000	0.00000	.00000	.00063	.00737	.02845	.06588	.11498	.17057	.22671	.28722	.34482	.40070

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[1] The ages shown here correspond to those in Table 3.9. Strictly cumulation is to exact ages one year greater than shown.

* Schedules used to obtain standard fertility at ages 10-24.

Table 3.10 cont'd.

			AGE					AGE					AGE		
SCHEDULE NUMBER	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	[illegible]	[illegible]	.49354	.51732	.58906	.62157	.66163	.70023	.73726	.77254	.80599	.83740	.86609	.89415	.91684
2	[illegible]	[illegible]	[illegible]	.61055	.65128	.69347	.73103	.76607	.79665	.82552	.85064	.88206	.90522	.92606	.94442
3	[illegible]	[illegible]	[illegible]	.55101	.57757	.62360	.66351	.70502	.74417	.78016	.81393	.84534	.87439	.90096	.92474
4	[illegible]	[illegible]	[illegible]	.56995	.59345	.63154	.67072	.70942	.74454	.77792	.81148	.84210	.87072	.89721	.92119
5	[illegible]	.51147	[illegible]	.62174	.66046	.70276	.73936	.77342	.80505	.83432	.86129	.88594	.90634	.92651	.94527
6	[illegible]	[illegible]	[illegible]	.56428	.59651	.63299	.67444	.71367	.75122	.78637	.81730	.84990	.87815	.90398	.92707
7	[illegible]	.47504	[illegible]	.56065	.60176	.64152	.67975	.71649	.75167	.78513	.81681	.84659	.87439	.90014	.92345
8	[illegible]	.52094	.56803	.61309	.65575	.69364	.73325	.76811	.80047	.83042	.83302	.88320	.90619	.92682	.94500
9	[illegible]	.53719	.49576	.53332	.57950	.62391	.66630	.70595	.74488	.78090	.81465	.84602	.87499	.90140	.92516
10	[illegible]	.52412	.56755	.60933	.64933	.68737	.72333	.75733	.78935	.81637	.84739	.87335	.89728	.91911	.93500
11	[illegible]	[illegible]	[illegible]	.65505	.69665	.70503	.74175	.77575	.80710	.83610	.86279	.88719	.90936	.92931	.94587
12	[illegible]	[illegible]	[illegible]	.51750	.57413	.61638	.65712	.69612	.73391	.76970	.80361	.83549	.86530	.89290	.91739
13	[illegible]	[illegible]	[illegible]	.55405	.60548	.64494	.68277	.71921	.75413	.79127	.81064	.84012	.87509	.90115	.92422
14	.47444	.52400	.57115	.61613	.65289	.67958	.73575	.77030	.80240	.83214	.85740	.88445	.90717	.92750	.94559
15	[illegible]	.43511	.49536	.57144	.58001	.62604	.66726	.70768	.74593	.78192	.81360	.84587	.87572	.90209	.92505
16	[illegible]	.49313	.53521	.57624	.61017	.65649	.69141	.72645	.76077	.79302	.82355	.85223	.87902	.90382	.92527
17	[illegible]	.53904	.58501	.62475	.67006	.70572	.74471	.77818	.80922	.83790	.86432	.88545	.91037	.93010	.94747
18	[illegible]	[illegible]	[illegible]	.54460	.57713	.61927	.65793	.69882	.73617	.77170	.80535	.83691	.86553	.89344	.91806
19	[illegible]	[illegible]	[illegible]	.56302	.59220	.69960	.73352	.76627	.79711	.82603	.85302	.87603	.90107	.92209	.94080
20	[illegible]	[illegible]	.50347	.54715	.59900	.63363	.67067	.70794	.74420	.77498	.81132	.84198	.87004	.89716	.92110
21	[illegible]	[illegible]	.50437	.54577	.64049	.68245	.72171	.75901	.79182	.82310	.85190	.87823	.90215	.92359	.94205
22	[illegible]	.49750	.53436	.56009	.61444	.65753	.69912	.72925	.76290	.79496	.82510	.85353	.88009	.90467	.92692
23	[illegible]	[illegible]	[illegible]	.63099	.67159	.71187	.74749	.78081	.81131	.83469	.86501	.88907	.91136	.93087	.94805
24	[illegible]	.46094	.49800	.53121	.58057	.62340	.66279	.70150	.73655	.77377	.80715	.83549	.86718	.89448	.91942
25	[illegible]	.44061	.53115	.57953	.61367	.65144	.69868	.72444	.75686	.79119	.82199	.85092	.87790	.90298	.92552
26	[illegible]	.53244	.57914	.62302	.66554	.70676	.74125	.77519	.80605	.83573	.86299	.88595	.90917	.92916	.94676
27	[illegible]	.43002	.51726	.56612	.58319	.62409	.67094	.71103	.74906	.78478	.81314	.84906	.87756	.90357	.92680
28	[illegible]	.50140	.54674	.58313	.62228	.65909	.69640	.73121	.76465	.79537	.82640	.85402	.88099	.90538	.92747
29	.50075	.54938	.59358	.63650	.67700	.71480	.75013	.78240	.81328	.84137	.86722	.89003	.91229	.93159	.94859
30	.40442	.45122	.49902	.54119	.58425	.62585	.66542	.70420	.74094	.77586	.80092	.83997	.86999	.89585	.92016
31	.48329	.52929	.57137	.61209	.65252	.69023	.72565	.75457	.79130	.82109	.84801	.87453	.89823	.91946	.93917
32	.42244	.46795	.51213	.55518	.59695	.63724	.67009	.71325	.74888	.78272	.81476	.84487	.87300	.89904	.92200
33	.45546	.50650	.55542	.62209	.66643	.68832	.72055	.76261	.79588	.82658	.85484	.88065	.90411	.92522	.94380

Table 3.10 cont'd.

SCHEDULE			AGE					AGE		
NUMBER	40	41	42	43	44	45	46	47	48	49
• 1	.94033	.95909	.97222	.98274	.98991	.99438	.99720	.99888	.99976	1.00000
2	.95945	.97244	.98211	.98710	.99173	.99655	.99830	.99932	.99986	1.00000
• 3	.95519	.96195	.97497	.98459	.99136	.99503	.99754	.99902	.99979	1.00000
• 4	.94217	.95931	.97306	.98125	.99020	.99455	.99728	.99892	.99977	1.00000
5	.96139	.97315	.98271	.98946	.99334	.99607	.99845	.99935	.99986	1.00000
• 6	.94869	.96307	.97576	.98509	.99135	.99521	.99762	.99905	.99950	1.00000
• 7	.96373	.96089	.97981	.98373	.99149	.99470	.99715	.99895	.99977	1.00000
8	.95637	.97273	.98230	.98821	.99160	.99459	.99731	.99911	.99980	1.00000
• 9	.94590	.96210	.97513	.98609	.99112	.99509	.99755	.99903	.99979	1.00000
10	.95531	.96694	.97462	.98766	.99273	.99594	.99800	.99921	.99943	1.00000
11	.96173	.97367	.98290	.98959	.99691	.99671	.99837	.99935	.99950	1.00000
•12	.94961	.95761	.97191	.98255	.99479	.99432	.99717	.99887	.99976	1.00000
•13	.96410	.95048	.97408	.98390	.99058	.99476	.99719	.99895	.99979	1.00000
14	.96080	.97303	.98249	.98033	.99386	.99603	.99833	.99936	.99985	1.00000
•15	.96547	.96230	.97531	.98481	.99114	.99512	.99759	.99906	.99979	1.00000
16	.94590	.95196	.97678	.98433	.99014	.99640	.99746	.99899	.99978	1.00000
17	.96215	.97345	.98399	.98870	.99408	.99574	.99839	.99936	.99950	1.00000
•18	.96020	.97001	.97717	.98271	.99069	.99437	.99720	.99889	.99976	1.00000
19	.96836	.97008	.98035	.98793	.99300	.99613	.99938	.99924	.99984	1.00000
•20	.96205	.95930	.97303	.98125	.99020	.99455	.99728	.99892	.99977	1.00000
21	.95644	.97157	.98154	.98376	.99353	.99544	.99824	.99930	.99985	1.00000
22	.96823	.96224	.97500	.98647	.99092	.99495	.99749	.99900	.99979	1.00000
23	.95257	.97423	.98329	.98982	.99414	.99574	.99841	.99937	.99987	1.00000
•24	.96075	.95590	.97244	.98259	.99434	.99443	.99722	.99889	.99976	1.00000
25	.96532	.96181	.97678	.98629	.99076	.99486	.99744	.99894	.99979	1.00000
26	.96165	.97361	.98237	.98976	.99609	.99670	.99817	.99935	.99990	1.00000
•27	.96673	.96291	.97571	.98505	.99113	.99528	.99752	.99905	.99980	1.00000
•28	.96684	.95256	.97519	.98459	.99034	.99498	.99750	.99900	.99979	1.00000
29	.96237	.97452	.98109	.98592	.99420	.99681	.99843	.99935	.99987	1.00000
•30	.96131	.95974	.97269	.98303	.99098	.99440	.99725	.99890	.99977	1.00000
31	.95573	.96923	.97961	.98758	.99280	.99602	.99802	.99921	.99983	1.00000
•32	.96311	.96005	.97353	.98355	.99038	.99465	.99733	.99894	.99977	1.00000
33	.95951	.97214	.98191	.98899	.99365	.99652	.99826	.99932	.99985	1.00000

+ Schedules used to obtain standard fertility at ages 40-49.

Table 3.11: Y values by single years[1] for the 33 schedules used to calculate standard fertility

SCHEDULE NUMBER	AGE 10	11	12	13	14
1	[illegible]	[illegible]	-1.89062	-1.68431	-1.49628
•2	-2.55602	-2.12115	-1.86375	-1.65114	-1.45016
3	-2.51542	-2.18166	[illegible]	-1.75541	-1.57665
4	-2.71423	-2.23144	-1.92679	-1.69963	-1.49599
•5	-2.76623	[illegible]	-1.89660	-1.66752	-1.45792
6	[illegible]	[illegible]	-1.94205	-1.74473	-1.59331
•7	**********	-2.22930	-1.86849	-1.71565	-1.49355
•8	**********	[illegible]	-1.94727	-1.75252	-1.53244
9	**********	-2.35076	-2.07674	-1.88021	-1.66244
•10	**********	[illegible]	-2.01673	-1.71535	-1.46392
•11	**********	-2.45541	-2.06666	-1.75925	-1.54032
12	**********	[illegible]	-2.13015	-1.87455	-1.64953
•13	**********	**********	-2.11475	-1.85045	-1.57437
•14	**********	**********	-2.21106	-1.89437	-1.63516
15	**********	**********	-2.27363	-2.00367	-1.77447
•16	**********	**********	[illegible]	-1.89632	-1.57135
•17	**********	**********	-2.37491	-1.94693	-1.65521
18	**********	**********	-2.41420	[illegible]	-1.77155
•19	**********	**********	**********	-1.94578	-1.52665
20	**********	**********	**********	-2.15167	-1.61128
21	**********	**********	**********	-2.17716	-1.85379
•22	**********	**********	**********	-2.23042	-1.70758
•23	**********	**********	**********	-2.30067	-1.83342
24	**********	**********	**********	-2.36237	-1.95426
•25	**********	**********	**********	**********	-1.92375
•26	**********	**********	**********	**********	-2.02723
27	**********	**********	**********	**********	-2.15734
28	**********	**********	**********	**********	-2.02046
•29	**********	**********	**********	**********	-2.19240
30	**********	**********	**********	**********	-2.29302
•31	**********	**********	**********	**********	**********
32	**********	**********	**********	**********	**********
33	**********	**********	**********	**********	**********

SCHEDULE NUMBER	AGE 15	16	17	18	19
1	-1.31978	-1.15099	-.98878	-.83004	-.69366
•2	-1.27173	-1.09394	-.92063	-.75103	-.58866
3	-1.40528	-1.23873	-1.07570	-.91557	-.76229
4	-1.30951	-1.13267	-.96440	-.80257	-.65068
•5	-1.20229	-1.07615	-.89689	-.72272	-.55709
6	-1.41275	-1.23011	-1.06865	-.90233	-.74535
•7	-1.29309	-1.10750	-.93315	-.76753	-.61332
•8	-1.32865	-1.13535	-.95837	-.77151	-.60212
9	-1.47570	-1.29018	-1.12237	-.95331	-.79276
•10	-1.24291	-1.04274	-.85735	-.68248	-.52004
•11	-1.31923	-1.11940	-.92145	-.73720	-.56430
12	-1.44417	-1.25245	-1.07178	-.89938	-.74004
•13	-1.34475	-1.13718	-.94815	-.77267	-.61201
•14	-1.40031	-1.18973	-.99332	-.79240	-.61428
15	-1.56615	-1.36596	-1.17724	-.99620	-.82620
•16	-1.31129	-1.09116	-.89689	-.72001	-.55992
•17	-1.39194	-1.15073	-.94676	-.74959	-.56807
18	-1.52942	-1.31140	-1.11160	-.92597	-.75599
•19	-1.23015	-.99010	-.79610	-.61576	-.45245
20	-1.53243	-1.29271	-1.07454	-.88597	-.71181
21	-1.59971	-1.36796	-1.12613	-.91913	-.72850
•22	-1.38310	-1.10469	-.89300	-.70833	-.54515
•23	-1.48942	-1.21008	-.97057	-.75716	-.58605
24	-1.66061	-1.38548	-1.15711	-.95225	-.76973
•25	-1.49959	-1.19948	-.96611	-.76490	-.59241
•26	-1.63105	-1.31834	-1.05693	-.82833	-.62678
27	-1.83735	-1.56570	-1.32305	-1.10181	-.90224
28	-1.41904	-1.10174	-.87630	-.68951	-.52725
•29	-1.66023	-1.26934	-.98430	-.75535	-.55541
30	-1.82697	-1.48471	-1.20951	-.97718	-.77673
•31	-1.62682	-1.19189	-.91924	-.70588	-.52529
32	-1.89328	-1.45738	-1.14851	-.90653	-.70785
33	-1.99765	-1.59126	-1.26970	-1.00000	-.77149

[1] The ages shown here correspond to those in Table 3.9. Strictly Y refers to exact ages one year greater than shown.

* Schedules used to obtain standard fertility at ages 10-24.

Table 3.11 cont'd.

SCHEDULE NUMBER			AGE		
	20	21	22	23	24
1	-.54009	-.40659	-.27712	-.15079	-.02627
*2	-.33328	-.28420	-.13465	.00187	.19141
3	-.61572	-.47640	-.33402	-.29391	-.07118
4	-.50793	-.37233	-.24207	-.11531	.00937
*5	-.34062	-.29675	-.10313	.03907	.17902
6	-.55537	-.45175	-.31282	-.17701	-.04314
*7	-.46929	-.33311	-.20274	-.07019	.04806
*8	-.44135	-.28812	-.14534	.00371	.14526
9	-.54021	-.44440	-.35353	-.21600	-.00060
*10	-.36421	-.22647	-.08758	.04569	.17049
*11	-.40137	-.26061	-.09417	.04540	.13737
12	-.54070	-.46439	-.31465	-.18388	-.05571
*13	-.46375	-.32861	-.19266	-.06693	.05941
*14	-.46715	-.28424	-.13634	.00465	.13144
15	-.56517	-.51432	-.35457	-.22708	-.00643
*16	-.41321	-.27862	-.14614	-.02052	.10249
*17	-.39031	-.28045	-.08942	.05506	.13866
18	-.53040	-.47317	-.31442	-.14085	-.05094
*19	-.40251	-.18151	-.02645	.10330	.23211
20	-.55424	-.30622	-.26709	-.13455	-.00537
21	-.55104	-.38559	-.22752	-.07578	.07222
*22	-.34751	-.26034	-.13140	-.00081	.11612
*23	-.35170	-.23010	-.07729	.06952	.21251
24	-.50083	-.45206	-.31642	-.17419	-.04228
*25	-.43848	-.29704	-.18419	-.01565	.08770
*26	-.46694	-.27714	-.12070	.02144	.17507
27	-.72021	-.35104	-.34358	-.24252	-.09052
28	-.41111	-.24554	-.11719	.00693	.12637
*29	-.37726	-.21427	-.06121	.00541	.22749
30	-.60450	-.44707	-.30110	-.16365	-.03061
*31	-.36450	-.21658	-.07700	.05736	.18857
32	-.53680	-.38334	-.24203	-.10794	.02145
33	-.57018	-.34606	-.27110	-.06274	.08433

*Schedules used to obtain standard fertility at

		AGE		
25	26	27	28	29
.09778	.22233	.34805	.47616	.60766
.28079	.92090	.58234	.70045	.85389
.08047	.19410	.32884	.48508	.60610
.13338	.25770	.38321	.51097	.64209
.31847	.45009	.60802	.74397	.89118
.09027	.22430	.35443	.44697	.63777
.17151	.29525	.42001	.54707	.61749
.28509	.42752	.50991	.71478	.80281
.05419	.19945	.32567	.48420	.60588
.30845	.43090	.58842	.70241	.83977
.32792	.45894	.51084	.75518	.90256
.07132	.19030	.32818	.45599	.58695
.18374	.30763	.43741	.55990	.85908
.24410	.43054	.57488	.72506	.87348
.04902	.18049	.32453	.48456	.60749
.22449	.34875	.47007	.59574	.72404
.33949	.48132	.62383	.76765	.91533
.07792	.20007	.33485	.48505	.59840
.35988	.40325	.51790	.75033	.89626
.12211	.24911	.37858	.50501	.63800
.21013	.30411	.51010	.65797	.80051
.23773	.35943	.48228	.60746	.73517
.15392	.49524	.53717	.78134	.92055
.08739	.21023	.34523	.47545	.60928
.21075	.33301	.45778	.50395	.71330
.31847	.40155	.50492	.75031	.89855
.04881	.18047	.32991	.47235	.61755
.28411	.37025	.49280	.61741	.74578
.35857	.50940	.65048	.79459	.94141
.09947	.22842	.35734	.49777	.62091
.31805	.44899	.38032	.71406	.85117
.14857	.27497	.40169	.53023	.66179
.23798	.30532	.53219	.68051	.83122

ages 10-24.

Table 3.11 cont'd.

SCHEDULE NUMBER	AGE 30	31	32	33	34
1	.76439	.98620	1.03119	1.15406	1.35453
2	1.00694	1.16056	1.32267	1.61234	1.57262
3	.75006	.99957	1.05522	1.21913	1.34330
4	.77761	.91740	1.08601	1.22082	1.33689
5	1.04201	1.19733	1.35803	1.52456	1.70652
6	.76206	.93173	1.06753	1.25155	1.42577
7	.91212	.95196	1.09665	1.25363	1.41920
8	1.01436	1.17631	1.43266	1.50257	1.68246
9	.75131	.90137	1.05779	1.22239	1.34712
10	.93117	1.12747	1.28028	1.64161	1.61373
11	1.06405	1.20497	1.37072	1.56035	1.72040
12	.72291	.86775	1.01627	1.17324	1.34034
13	.92618	.96371	1.11012	1.26715	1.43050
14	1.02565	1.19136	1.34151	1.51177	1.59419
15	.74955	.90656	1.05134	1.22718	1.40239
16	.85826	.94048	1.14265	1.29174	1.45141
17	1.00566	1.22159	1.45312	1.57263	1.73245
18	.71961	.87702	1.02622	1.19321	1.35033
19	1.06635	1.17149	1.37151	1.48306	1.65482
20	.77622	.91535	1.06408	1.21929	1.39566
21	.96215	1.11947	1.28352	1.45490	1.63518
22	.88926	1.00735	1.15276	1.30089	1.47135
23	1.07922	1.23430	1.34580	1.56490	1.74652
24	.76599	.86943	1.03633	1.17332	1.35079
25	.86729	.98823	1.13208	1.28592	1.45151
26	1.05010	1.20572	1.30787	1.53779	1.71734
27	.76557	.91707	1.07576	1.26162	1.41737
28	.77855	1.01674	1.16169	1.31550	1.41478
29	1.05173	1.24650	1.40756	1.57661	1.75603
30	.75792	.99451	1.06786	1.20451	1.37126
31	.94224	1.13429	1.29049	1.45210	1.62364
32	.79732	.93727	1.08494	1.24064	1.40655
33	.98005	1.14244	1.30590	1.47707	1.65831

		AGE		
35	36	37	38	39
1.53345	1.72945	1.44602	2.19030	2.40929
1.66549	2.07559	2.30617	2.50040	2.88149
1.58044	1.75371	2.00817	2.26057	2.54800
1.56544	1.75106	1.97732	2.22129	2.50002
1.40162	2.11107	2.34271	2.60134	2.89625
1.61245	1.81023	2.04071	2.29313	2.50058
1.59775	1.74243	2.00825	2.25155	2.53024
1.47545	2.04623	2.31747	2.57706	2.87218
1.58475	1.76040	2.01330	2.26004	2.55376
1.79422	1.99943	2.22702	2.47277	2.75676
1.91337	2.12279	2.45366	2.01296	2.90783
1.52031	1.71028	1.93125	2.17737	2.45715
1.60838	1.80340	2.01908	2.26254	2.54022
1.68755	2.09742	2.12854	2.59819	2.88329
1.59944	1.74448	2.31401	2.27260	2.50035
1.63419	1.83319	2.94439	2.29144	2.55936
1.42540	2.13409	2.38545	2.62457	2.91945
1.53029	1.72621	1.99313	2.18770	2.45094
1.53912	2.03962	2.29165	2.51196	2.79757
1.56475	1.76029	1.97667	2.22077	2.49755
1.83040	2.04135	2.27331	2.53349	2.82911
1.64994	1.94276	2.15782	2.30074	2.57934
1.93731	2.14094	2.37710	2.63616	2.93054
1.54061	1.73041	1.95320	2.19765	2.47678
1.62949	1.82365	2.03901	2.28219	2.56022
1.91104	2.12054	2.35157	2.61006	2.90578
1.60542	1.81017	2.03553	2.28809	2.57678
1.65721	1.85009	2.06552	2.30063	2.50633
1.94864	2.15762	2.38814	2.64709	2.94167
1.55089	1.76050	1.90313	2.20744	2.48044
1.80037	2.00943	2.23189	2.48253	2.76044
1.58543	1.78040	1.99645	2.24028	2.51882
1.85251	2.06286	2.24458	2.55457	2.85003

Table 3.11 cont'd.

SCHEDULE NUMBER	AGE 40	41	42	43	44
• 1	2.76624	3.15196	3.56493	4.05086	4.54091
2	3.14739	3.57734	4.01429	4.51324	5.00849
• 3	2.87558	3.24720	3.67640	4.16647	4.71235
• 4	2.81474	3.18120	3.59074	4.08099	4.52099
5	3.23208	3.71205	4.00857	4.50764	5.10246
• 6	2.90431	3.27540	3.70769	4.19740	4.74543
• 7	2.84459	3.21072	3.62424	4.11061	4.65024
8	3.20911	3.57690	4.02593	4.52412	5.07911
• 9	2.85172	3.25344	3.68141	4.17170	4.71924
10	3.06932	3.45800	3.88312	4.37249	4.92019
11	3.26154	3.62150	4.06015	4.55416	5.11412
•12	2.77635	3.13918	3.55531	4.03446	4.57951
•13	2.85917	3.22136	3.63462	4.12070	4.66050
14	3.21914	3.59956	4.03697	4.53516	5.09017
•15	2.89471	3.26055	3.68872	4.17915	4.72635
16	2.88706	3.26431	3.65743	4.14637	4.69008
17	3.25509	3.63705	4.07161	4.57054	5.12572
•18	2.78609	3.16099	3.56789	4.04912	4.59917
19	3.12914	3.49914	3.92110	4.41073	4.95791
•20	2.81439	3.18040	3.59949	4.08077	4.62071
21	3.11545	3.50538	3.94244	4.43229	5.03754
22	2.89494	3.29536	3.67638	4.15727	4.79035
23	3.20801	3.58624	4.04240	4.54173	5.13695
•24	2.75526	3.15855	3.57731	4.05571	4.59571
25	2.87437	3.26035	3.65843	4.13951	4.67925
26	3.26152	3.62157	4.05420	4.55721	5.11213
•27	2.90905	3.27710	3.70527	4.19580	4.74360
•28	2.90820	3.26597	3.68108	4.16684	4.70449
29	3.27716	3.65947	4.09443	4.59233	5.14742
•30	2.86534	3.16803	3.58671	4.06805	4.60804
31	3.00880	3.46530	3.89257	4.35231	4.92954
•32	2.83741	3.19576	3.61121	4.09940	4.63927
33	3.18627	3.59403	4.00352	4.50273	5.05807

♦ Schedules used to obtain standard fertility a

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                                  AGE
     45          46          47           48          49
5.17694     5.87734     6.79709     8.33950***********
5.66814     6.37361     7.29857     8.84375***********
5.30507     6.00792     6.92449     8.47371***********
5.20900     5.90740     6.82715     8.36959***********
5.70251     6.40520     7.33297     8.87030***********
5.33928     6.04120     6.96344     8.50725***********
5.23831     5.93868     6.85642     8.39835***********
5.67903     6.38479     7.30952     8.85493***********
5.31316     6.01515     6.93739     8.48117***********
5.51394     6.21540     7.13912     8.68262***********
5.71600     6.41951     7.34440     8.88907***********
5.16771     5.86616     6.78596     8.32839***********
5.24155     5.94691     6.86864     8.40907***********
5.69007     6.39579     7.32057     8.86579***********
5.32056     6.02249     6.94521     8.48912***********
5.27607     5.97439     6.89411     8.43832***********
5.72538     6.43107     7.35583     8.90125***********
5.17737     5.87582     6.79561     8.33805***********
5.55142     6.25354     7.17573     8.71952***********
5.20884     5.90725     6.82702     8.36945***********
5.63732     6.34307     7.26788     8.81332***********
5.28493     5.98324     6.90245     8.44535***********
5.73650     6.44217     7.36694     8.91235***********
5.18691     5.88535     6.80516     8.34759***********
5.28726     5.98559     6.90532     8.44773***********
5.71208     6.41776     7.34253     8.88795***********
5.33765     6.03980     6.96213     8.50808***********
5.24245     5.94075     6.86045     8.40200***********
5.74708     6.45272     7.37748     8.92207***********
5.19620     5.89463     6.81441     8.35606***********
5.52332     6.22527     7.14748     8.69135***********
5.22731     5.92575     6.84551     8.38794***********
5.65778     6.36352     7.28832     8.83376***********
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t ages 40-49.

Table 3.12: Values of ΔY by single years for the 33 schedules used to calculate standard fertility.

[illegible]			AGE					AGE					AGE		
[illegible]	10-11	11-12	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24	24-25
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.15979	.16271	.15754	.14090	.14177	.13430	.12947	.12633	.12451	.12405
* 2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17724	.17315	.16466	.16238	.15518	.14900	.14455	.14152	.13957	.13927
3	[illegible]	[illegible]	[illegible]	[illegible]	.17117	.15555	.16313	.16312	.15260	.14558	.14092	.13578	.13111	.12552	.13235
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17672	.16497	.16143	.15169	.14275	.13560	.13026	.12676	.12467	.12401
* 5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.14511	.17795	.17417	.16552	.15768	.13067	.14552	.14220	.14945	.13945
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.17066	.16495	.15552	.15750	.14599	.14352	.13593	.13591	.13387	.13341
* 7	*******	[illegible]	[illegible]	[illegible]	[illegible]	.14551	.17435	.14963	.15920	.14403	.13318	.13037	.12655	.12425	.12345
* 8	*******	[illegible]	[illegible]	[illegible]	[illegible]	.14111	.13442	.17666	.16940	.16077	.15323	.14778	.14405	.14155	.14003
9	*******	[illegible]	[illegible]	[illegible]	[illegible]	.17652	.17331	.16905	.16055	.15253	.14303	.14007	.13753	.13540	.13479
* 10	*******	[illegible]	[illegible]	[illegible]	[illegible]	.21107	.18514	.17487	.16245	.15186	.14337	.13729	.13327	.14077	.12999
* 11	*******	[illegible]	[illegible]	[illegible]	[illegible]	.20963	.19255	.14425	.17291	.16297	.15452	.14044	.14427	.14147	.14055
12	*******	[illegible]	[illegible]	[illegible]	[illegible]	.19111	.16037	.17190	.15994	.14314	.14111	.13494	.13077	.12815	.12705
* 13	*******	*******	[illegible]	[illegible]	[illegible]	.20759	.16903	.17548	.16000	.14326	.13699	.13215	.12767	.12492	.12301
* 14	*******	*******	[illegible]	[illegible]	[illegible]	.21594	.20140	.19093	.17012	.16711	.15737	.15120	.14537	.14341	.14221
15	*******	*******	[illegible]	[illegible]	[illegible]	.18314	.15373	.16104	.16999	.16004	.15155	.14574	.14150	.13865	.13745
* 16	*******	*******	[illegible]	[illegible]	[illegible]	.22014	.19420	.17567	.16090	.14572	.13999	.13007	.12562	.12299	.12203
* 17	*******	*******	[illegible]	[illegible]	[illegible]	.21411	.21116	.19715	.16152	.15574	.15039	.15103	.14590	.14258	.14125
18	*******	*******	[illegible]	[illegible]	[illegible]	.21402	.19440	.19572	.16993	.15550	.14532	.13675	.13159	.14021	.12030
* 19	*******	*******	*******	[illegible]	[illegible]	.21096	.20040	.19014	.15311	.15133	.14101	.13465	.13065	.12831	.12773
20	*******	*******	*******	[illegible]	[illegible]	.20622	.21317	.14357	.17410	.15557	.14702	.13802	.13294	.12929	.12745
21	*******	*******	*******	[illegible]	[illegible]	.24171	.22113	.20099	.19003	.17704	.16567	.15777	.15204	.14800	.14011
* 22	*******	*******	*******	[illegible]	[illegible]	.25108	.21159	.19469	.16310	.14704	.13092	.12959	.12499	.12233	.12141
* 23	*******	*******	*******	[illegible]	[illegible]	.27174	.23401	.21151	.17111	.17435	.16161	.15201	.14590	.14247	.14133
24	*******	*******	*******	[illegible]	[illegible]	.26093	.22631	.20486	.16231	.16491	.15190	.14255	.13513	.13191	.12701
* 25	*******	*******	*******	*******	[illegible]	.30012	.23530	.19422	.17299	.15392	.14139	.13240	.12734	.12435	.12305
* 26	*******	*******	*******	*******	[illegible]	.31271	.25141	.22459	.20155	.16104	.16715	.15703	.15019	.14559	.14345
27	*******	*******	*******	*******	[illegible]	.27105	.24204	.22124	.19937	.16204	.16637	.15026	.15107	.14599	.14313
28	*******	*******	*******	*******	[illegible]	.31724	.22545	.19679	.16225	.14514	.13551	.12041	.12402	.12154	.12075
* 29	*******	*******	*******	*******	[illegible]	.37004	.26194	.23295	.19994	.17415	.16299	.15505	.14602	.14240	.14078
30	*******	*******	*******	*******	[illegible]	.34176	.27520	.23733	.19044	.17423	.15743	.14572	.13791	.13244	.13008
* 31	*******	*******	*******	*******	*******	.43243	.27235	.21336	.18059	.16479	.14792	.13950	.13430	.13121	.13007
32	*******	*******	*******	*******	*******	.43590	.30377	.24198	.19000	.17105	.15325	.14152	.13405	.12943	.12712
33	*******	*******	*******	*******	*******	.40840	.32135	.26910	.22911	.20131	.18132	.16770	.15842	.15206	.14000

* Schedules used to obtain standard fertility at ages 10-24.

Table 3.12 cont'd.

SCHEDULE NUMBER	AGE 25-26	26-27	27-28	28-29	29-30	30-31
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.14001
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.14012
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]	.13711	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
33	[illegible]	[illegible]	[illegible]	.15071	[illegible]	.15754

	AGE					AGE		
31-32	32-33	33-34	34-35	35-36	36-37	37-38	38-39	39-40
14734	.15822	.16046	.17443	.19550	.21657	.24428	.27899	.31695
[illegible]	.15991	.18024	.19339	.20969	.23116	.25954	.24509	.33589
[illegible]	.16342	.17415	.16714	. 0326	.22446	.25240	.28743	.32768
14772	.15581	.16006	.17405	.19514	.21624	.24398	.27872	.31673
[illegible]	.15452	.17996	.19311	.20944	.23095	.25933	.28491	.33575
[illegible]	.16452	.17422	.18718	.20329	.22447	.25242	.28745	.32773
[illegible]	.15322	.18352	.17855	.19468	.21562	.24360	.27639	.31645
16215	.17011	.18039	.19349	.20978	.23124	.25959	.24513	.33593
15044	.15450	.17473	.18764	.20370	.22404	.25274	.28773	.32796
[illegible]	.18139	.17172	.16489	.20120	.22200	.25075	.28501	.32654
[illegible]	.15483	.17994	.19308	.20940	.23090	.25928	.29487	.33571
[illegible]	.15697	.18710	.17998	.19597	.21597	.24462	.27928	.31920
[illegible]	.15546	.16534	.17818	.19452	.21507	.24347	.27627	.31535
[illegible]	.17018	.16041	.19348	.20976	.23121	.25455	.24510	.33590
15715	.16514	.17522	.19505	.20404	.22513	.25299	.28795	.32315
[illegible]	.15424	.16487	.17774	.19400	.21520	.24305	.27791	.31605
18153	.18651	.17962	.19295	.20929	.23079	.25918	.24478	.33564
[illegible]	.15701	.18710	.17995	.19593	.21692	.24457	.27923	.31915
[illegible]	.18045	.17095	.18421	.20060	.22206	.25028	.28561	.32021
14774	.15521	.16538	.17424	.19533	.21639	.24409	.27682	.31800
[illegible]	.17138	.18146	.19442	.21038	.23193	.24017	.24583	.33034
[illegible]	.15405	.16440	.17754	.19353	.21505	.24292	.27780	.31795
[illegible]	[illegible]	.17983	.19778	.20913	.23006	.25907	.29488	.33555
[illegible]	.15869	.16697	.17982	.19560	.21679	.24445	.27913	.31906
[illegible]	.15454	.18459	.17799	.19417	.21535	.24319	.27603	.31814
[illegible]	.16466	.16011	.19420	.20950	.23098	.25934	.29491	.33575
15807	.18829	.17575	.18445	.20434	.22536	.25317	.28809	.32627
[illegible]	.13384	.18426	.17743	.19368	.21493	.24281	.27770	.31767
[illegible]	.18407	.17942	.19260	.20898	.23052	.25895	.29458	.33547
[illegible]	.15965	.16675	.17962	.19561	.21663	.24431	.27900	.31696
15289	.18113	.17154	.18473	.20106	.22247	.25063	.24591	.32646
14717	.15570	.15591	.17888	.19496	.21606	.24380	.27858	.31959
[illegible]	.17117	.16124	.19414	.21036	.23172	.25999	.24546	.33620

AGE		
32-33	33-34	34-35
15877	.16040	.17443
15991	.16824	.19138
16192	.17415	.16714
15581	.15006	.17405
[illegible]	.17998	.19111
[illegible]	.17422	.18716
[illegible]	.18552	.17855
17011	.18039	.19349
[illegible]	.17473	.18706
18131	.17172	.16489
[illegible]	.17994	.19308
[illegible]	.16710	.17998
[illegible]	.16534	.17838
17018	.15041	.19348
16514	.17522	.15405
[illegible]	.16487	.17779
[illegible]	.17462	.19295
15701	.18710	.17995
18045	.17095	.18421
[illegible]	.15038	.17429
17138	.18140	.19442
[illegible]	.16440	.17759
[illegible]	.17983	.19718
[illegible]	.15097	.17482
[illegible]	.18459	.17799
[illegible]	.16011	.19128
[illegible]	.17575	.16845
15384	.18426	.17743
18407	.17942	.19280
15805	.18675	.17962
16113	.17154	.16473
15570	.15591	.17898
17117	.18124	.19419

		AGE		
35-36	36-37	37-38	38-39	39-40
.19550	.21657	.24426	.27899	.31695
.20969	.23110	.25934	.24509	.33069
. 0326	.22446	.25240	.24743	.32768
.19514	.21624	.24398	.27872	.31673
.20944	.23095	.25933	.24491	.33573
.20329	.22447	.25242	.24745	.32773
.19468	.21562	.24360	.27639	.31645
.20978	.23124	.25959	.24513	.33593
.20370	.22404	.25274	.24773	.32796
.20120	.22200	.25075	.24501	.32054
.20940	.23090	.25928	.24987	.33571
.19597	.21597	.24462	.27928	.31920
.19452	.21507	.24347	.27827	.31535
.20976	.23121	.25455	.24510	.33540
.20404	.22513	.25299	.24795	.32915
.19400	.21520	.24305	.27791	.31805
.20929	.23019	.25918	.24478	.33584
.19593	.21692	.24457	.27923	.31915
.20060	.22206	.25028	.24561	.32021
.19533	.21639	.24409	.27882	.31800
.21058	.23193	.25017	.24583	.33034
.19553	.21505	.24292	.27780	.31795
.20913	.23006	.25407	.24488	.33555
.19560	.21679	.24445	.27913	.31900
.19417	.21555	.24319	.27803	.31814
.20950	.23098	.25934	.24491	.33575
.20434	.22536	.25317	.24809	.32027
.19368	.21493	.24201	.27770	.31787
.20898	.23052	.25895	.24958	.33547
.19561	.21603	.24431	.27900	.31096
.20106	.22247	.25003	.24591	.32640
.19496	.21606	.24360	.27858	.31559
.21036	.23172	.25999	.24540	.33020

Table 3.12 cont'd.

SCHEDULE NUMBER	AGE						
	25-26	26-27	27-28	28-29	29-30	30-31	31-32
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.14045	.14717
33	[illegible]	[illegible]	[illegible]	.15071	[illegible]	.15754	.16340

Table 3.12 cont'd.

Schedule Number	40-41	41-42	42-43	43-44	44-45	45-46	46-47	47-48	48-49	49-50
+ 1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+ 3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+ 4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+ 6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+ 7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+ 9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+28	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
29	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+30	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
31	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
+32	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********
33	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	********	********

+ Schedules used to obtain standard fertility at ages 40-49.

Table 3.13: Average and standard values of ΔY and standard Y values by single years of age

Age	Adjusted average ΔY	Standard ΔY_s	$Y_s(x)$	Exact age x
10-11	∞	∞	-3.18852	11
11-12	.49270	.48844	-2.70008	12
12-13	.32998	.32713	-2.37295	13
13-14	.30295	.30033	-2.07262	14
14-15	.30217	.29956	-1.77306	15
15-16	.28611	.28020	-1.49286	16
16-17	.24783	.24225	-1.25061	17
17-18	.21108	.20582	-1.04479	18
18-19	.19061	.18552	-0.85927	19
19-20	.17290	.16797	-0.69130	20
20-21	.15944	.15805	-0.53325	21
21-22	.14931	.14801	-0.38524	22
22-23	.14224	.14101	-0.24423	23
23-24	.13759	.13640	-0.10783	24
24-25	.13463	.13347	0.02564	25
25-26	.13289	.13289	0.15853	26
26-27	.13294	.13294	0.29147	27
27-28	.13368	.13368	0.42515	28
28-29	.13586	.13586	0.56101	29
29-30	.13899	.13899	0.70000	30
30-31	.14272	.14272	0.84272	31
31-32	.14742	.14742	0.99014	32
32-33	.15393	.15393	1.14407	33
33-34	.16220	.16220	1.30627	34
34-35	.17245	.17245	1.47872	35
35-36	.18554	.18554	1.66426	36
36-37	.20171	.20171	1.86597	37
37-38	.22297	.22297	2.08894	38
38-39	.25099	.25099	2.33992	39
39-40	.28610	.28610	2.62602	40
40-41	.32162	.32898	2.95500	41
41-42	.36537	.37373	3.32873	42
42-43	.42148	.43111	3.75984	43
43-44	.48408	.49515	4.25499	44
44-45	.54232	.55471	4.80970	45
45-46	.58992	.60341	5.41311	46
46-47	.69953	.71553	6.12864	47
47-48	.92053	.94158	7.07022	48
48-49	1.54288	1.57817	8.64839	49
49-50	∞	∞	∞	50

$$\overline{F}_s(x \text{ to } x+4) = F_s(x) + \frac{1}{5}(4.5\ f_s(x) + 3.5\ f_s(x+1) + 2.5\ f_s(x+2) + 1.5\ f_s(x+3) + 0.5\ f_s(x+4)\ )$$

where $\overline{F}_s(x \text{ to } x+4)$ is average cumulative fertility (or average parity) for women aged x to x+4, $F_s(x)$ is cumulative fertility at exact age x where x = 15, 20, ... etc., and $f_s(x)$ is age specific fertility for the single year of age, x.

These average parities are obviously not equal to actual standard parity at ages 12.5, 17.5 , etc. because of the curvature of the fertility function, especially at very young and very old ages. They do not therefore refer to the exact midpoints of the age groups, but rather to the ages at which average and actual parities are equal. Estimation of these ages involves the interpolation of actual fertility between the single year values. For purposes of developing and using the transformed standard, however, knowledge of the exact ages is not necessary. Average parity is transformed to the appropriate Y_s value in the usual way, and it is these values of Y_s that constitute the "midpoint" standard, given in Table 3.14.

Standard fertility

Though in practice standard fertility is only of interest in its transformed form, $Y_s(x)$, it is pertinent to consider its more understandable forms, $f_s(x)$

Table 3.14: Five-year average standard parities and midpoint Y_s values

Age group	Average parity	Midpoint Y_s value
10-14	.00035	-2.07330
15-19	.05279	-1.07889
20-24	.25513	-0.31188
25-29	.49559	0.35380
30-34	.70644	1.05695
35-39	.86781	1.95343
40-44	.96760	3.41302
45-49	.99766	6.05569

and $F_s(x)$. These schedules appear by single years and by five year age groups in Table 3.15. Only the pattern of fertility is available, giving no indication of a standard level of fertility.

Calculation of the mean and standard deviation of $f_s(x)$ obviously depends on the rather rough value of 0.7 for $Y_s(30)$. Their values, $\mu_s = 28.29$ years and $\sigma_s = 7.25$, are of interest, however, for comparative purposes. The Coale-Trussell model parameters for the five-year standard values, estimated by the method described in Appendix 3.1 Section C where the standard appears as an example, are $a_0 = 12.54$, $k = 0.46$ and $m = 0.32$. The value of SMAM is 17.73. Again, these values are dependent on the value chosen for $Y_s(30)$.

The Gompertz fit to standard fertility

The transformed Gompertz model, described in Chapter 2, relates observed fertility to standard fertility. The iterative procedure used to estimate the parameters of the transformed Gompertz model (Chapter 5) requires initial estimates of these parameters. Clearly, the better the initial estimates, the more efficient the estimation procedure. These estimates are obtained from estimates of the ordinary Gompertz parameters, and it is therefore desirable to fit the ordinary Gompertz to standard fertility as well as possible. This is done

Table 3.15: Age specific and cumulative standard fertility schedules

	Age specific fertility		Cumulative fertility	
Age	Single year	5 year group	Single year	5 year endpoint
10-11	.00000		.00000	
11-12	.00000		.00000	
12-13	.00002		.00002	
13-14	.00033		.00035	
14-15	.00242	.00277	.00277	.00277
15-16	.00891		.01168	
16-17	.01875		.03043	
17-18	.02783		.05826	
18-19	.03602		.09428	
19-20	.04156	.13307	.13584	.13584
20-21	.04603		.18187	
21-22	.04806		.22993	
22-23	.04904		.27897	
23-24	.04932		.32829	
24-25	.04902	.24147	.37731	.37731
25-26	.04866		.42597	
26-27	.04774		.47371	
27-28	.04642		.52013	
28-29	.04504		.56517	
29-30	.04344	.23130	.60861	.60861
30-31	.04155		.65016	
31-32	.03952		.68968	
32-33	.03754		.72722	
33-34	.03553		.76275	
34-35	.03343	.18757	.79618	.79618
35-36	.03133		.82751	
36-37	.02912		.85663	
37-38	.02691		.88354	
38-39	.02462		.90816	
39-40	.02203	.13401	.93019	.93019
40-41	.01906		.94925	
41-42	.01555		.96480	
42-43	.01218		.97698	
43-44	.00893		.98591	
44-45	.00597	.06169	.99188	.99188
45-46	.00367		.99555	
46-47	.00227		.99782	
47-48	.00133		.99915	
48-49	.00067		.99982	
49-50	.00018	.00812	1.00000	1.00000

in Appendix 3.3, where the value of x_0, the origin of the age scale, that permits the best fit is also determined. The resulting fit is given by

$$\hat{F}_s(x) = \hat{F}_s \, C^{D^{x-x_0}} = 1.05374 \, . \, 0.04808^{0.8748^{x-16.732}}$$

Though this is the best fit in the least squares sense, it is seen in Appendix Table A3.3.1 that the Gompertz function is not a particularly good approximation to standard fertility. This serves to illustrate the fact that there is scope for improvement. By using the standard pattern of fertility to modify the Gompertz function this improvement can be partially realised.

CHAPTER 4

THE SIMULATION OF TEST DATA

Introduction

A common problem involved in the formulation of demographic models is the absence of good data to provide a measure of the model's validity. Such a problem arose in the development of this model, and it was therefore decided to provide data by simulation. A series of age-specific fertility rates typical of high fertility populations undergoing fertility decline was simulated using the Barrett simulation model, modified to meet requirements.

The Barrett Simulation Model

Descriptions of earlier forms of this Monte Carlo simulation model have been reported by Barrett (1967, 1969). The version used to produce fertility schedules for the present purpose was based on the form of the model described by Barrett (1971) and incorporating some of the more recent modifications reported in Barrett and Brass (1974). A description of this version of the model is included here, so that additional modifications can be described adequately. A compact description of

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Marriage

The model simulates the individual birth histories of a cohort of women, from marriage to the end of childbearing. The rates produced are therefore age specific marital fertility rates. Age at marriage is an input parameter, but is fixed in that all women in a cohort are assumed to marry at exactly the same age. The model does not allow for ages at marriage of less than 20 years, nor does it allow for marital dissolution before the end of childbearing. Problems of differing marriage duration, teenage pregnancies, illegitimacy, divorce, widowhood, separation and remarriage are therefore avoided.

Susceptible state

After marriage, events may occur at intervals of one lunar month (i.e. 28 days, hereafter referred to as a month) and are determined by monthly probabilities of occurrence. The first event that may take place is conception, since all women are assumed to be susceptible to conception from marriage. Each woman has a basic monthly probability of conception, ρ^*, also termed her fecundability, which is determined randomly from a beta distribution of fecundability among women:

$$f(\rho) = \frac{\rho^{a-1} \; (1-\rho)^{b-1} \; d\rho}{\beta(a,b)} \qquad 0 < \rho < 1 \qquad 4.1$$

where $\beta(a,b) = \int_0^1 x^{a-1} \; (1-x)^{b-1} \; dx$

Each woman's fecundability is determined by the generation of a series of a + b - 1 random numbers between 0 and 1, the a-th in magnitude of which is taken as ρ^*. The mean fecundability is a/a+b with variance

$$\frac{ab}{(a+b)^2(a+b+1)} \, .$$

The parameters of this distribution are a = 3 and b = 13, chosen to produce a similar completed family size distribution to women who had married at ages 20-24 and who were enumerated in the 1911 Census of Ireland (on which the model is largely based), given the way in which fecundability is allowed to tail off at older ages.

The basic monthly probability of conception applies to noncontracepting women from marriage to age 30. (For the effects of contraception on fecundability, see later.) Thereafter there is a monotonic and almost linear decline in fecundability from ρ^* at age 30, to 0 at a predetermined age at end of childbearing period (see below). The pertinent value of ρ is redetermined at the beginning of each period of susceptibility to conception, or at intervals of 2 years if conception does not occur within that length of time.

Given the probability of conception of a woman (whether she be contracepting or not) time to conception follows a geometric distribution

$$P\text{ (time to conception} = n) = \rho(1-\rho)^n \qquad n = 0, 1, 2 \ldots \text{ months}$$
$$= 0 \text{ otherwise}$$

with mean $\frac{1-\rho}{\rho}$ months and variance $\frac{1-\rho}{\rho^2}$ months^2.

An individual woman's time to conception is determined by

$$n = \frac{\ln(z)}{\ln(1-\rho)}$$

where z is a random number between 0 and 1. Values for n for given values of ρ and z are shown in Table 4.1.

Outcome of pregnancy

After conception, there are three possible events: foetal death, stillbirth and live birth, with probabilities θ_1, θ_2 and θ_3 respectively. The probabilities of foetal death and stillbirth increase linearly with age:

Table 4.1: Months* to conception for given levels of fecundability

Random number	Fecundability .01	.05	.10	.15	.20	.25	.30	.40	.50
.01	458	89	43	28	20	16	12	9	6
.1	229	44	21	14	10	8	6	4	3
.2	160	31	15	9	7	5	4	3	2
.3	119	23	11	7	5	4	3	2	1
.4	91	17	8	5	4	3	2	1	1
.5	68	13	6	4	3	2	1	1	1
.6	50	9	4	3	2	1	1	1	0
.7	35	6	3	2	1	1	1	0	0
.8	22	4	2	1	1	0	0	0	0
.9	10	2	1	0	0	0	0	0	0
.99	1	0	0	0	0	0	0	0	0

* The figures given are in fact obtained by truncating the fractional part rather than taking the nearest whole number. For example, 5.4 and 5.7 are both taken as 5 months.

Where $n > 26$, ρ is redetermined (if necessary) and another random number is generated. The resulting value of n is added to 26.

$$\theta_1 = 0.24 + 0.005\ (x - 30)$$

$$\theta_2 = 0.03 + 0.001\ (x - 30)$$

resulting in a corresponding decrease in the probability of a live birth ($\theta_3 = 1 - \theta_1 - \theta_2$). Table 4.2 gives these probabilities by age.

In the event of foetal death, the gestation period is distributed geometrically with monthly probability of foetal death equal to $0.11(0.55)^{n-2}$ where $2 \leq n \leq 8$ months. Losses in the first month of pregnancy are considered as reduced fecundability, and are already incorporated into the basic monthly probability of conception.

In the events of stillbirth and livebirth, the duration of pregnancy is fixed at 9 and 10 (lunar) months respectively. Insusceptibility to conception after foetal death and stillbirth is also fixed at 2 and 3 months respectively. After a livebirth however, the period of post partum insusceptibility is determined randomly. The interval is made up of a fixed one month delay plus two consecutive geometrically distributed delays with the same parameter. For noncontracepting women, this parameter is $r = 1/6$ with a mean delay to susceptibility to conception of $1.0 + 2(1-r)/r = 11$ months and a variance of 60 months^2. For contracepting women, see below.

Table 4.2: Probabilities of foetal death, stillbirth and livebirth by age

Age	Probability of foetal death	stillbirth	livebirth
20	.190	.020	.79
25	.215	.025	.76
30	.240	.030	.73
35	.265	.035	.70
40	.290	.040	.67
45	.315	.045	.64
50	.340	.050	.61

Sterility and menopause

The age at which conception can no longer take place can be determined by one of two random variables, age at sterility and age at menopause. The younger of these two ages is taken as marking the end of a woman's possible childbearing period.

Age at menopause follows a beta distribution between ages 38 and 54 with a mean of 47.6 years:

$$f(x) = \frac{(x-38)^2(54-x)}{K}$$ where K is the appropriate constant such that $\int_{38}^{54} f(x)dx = 1.0$

This function is tabulated in Table 4.3: individual ages at menopause are determined by the generation of a random number between 0 and 1.

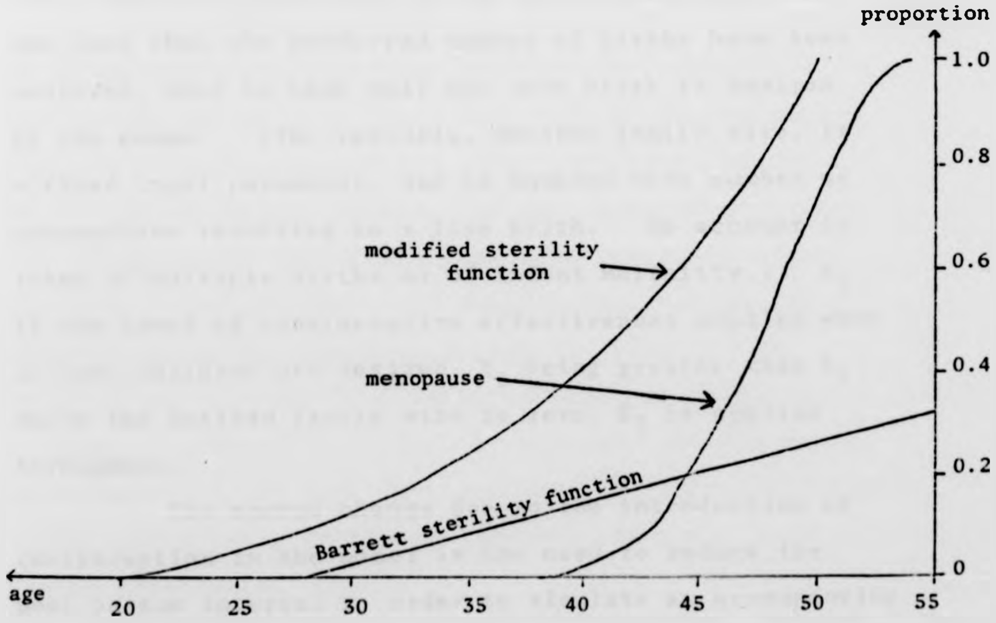
Age at sterility, where it occurs before menopause, is determined as 28 + z/0.012 years where z is a random variable between 0 and 1. At ages less than 28, a constant 4.8 per cent of women are assumed sterile.

The combined effect of these two functions is illustrated in Figure 4.1. The mean age of the end of the childbearing period is 43 years, and the median 46 years.

Table 4.3: Frequency and cumulative distributions of age at menopause

Age	f(x)	F(x)
38	0.000	0.000
39	.001	.001
40	.006	.007
41	.016	.023
42	.028	.051
43	.045	.096
44	.064	.160
45	.073	.233
46	.087	.320
47	.099	.419
48	.107	.526
49	.111	.637
50	.109	.746
51	.100	.846
52	.083	.929
53	.057	.986
54	.014	1.000

Figure 4.1: Cumulative distributions of age at menopause and age at sterility in the Barrett simulation model and in the modified Barrett simulation model.



Contracepting women

The introduction of contraception into the model requires changes to two functions. Firstly, the fecundability function is modified by multiplying ρ by a factor 1 - E, where E is a measure of use-effectiveness of contraception. The model allows for two levels of contraceptive effectiveness, E_1 and E_2. E_1 is the level applied in order to try to attain a last birth interval of two years. It is therefore applied for two years following the start of the fecund interval when one less than the preferred number of births have been achieved, that is when only one more birth is desired by the woman. (The variable, desired family size, is a fixed input parameter, and is equated with number of conceptions resulting in a live birth. No account is taken of multiple births or of infant mortality.) E_2 is the level of contraceptive effectiveness applied when no more children are desired, E_2 being greater than E_1. Where the desired family size is zero, E_2 is applied throughout.

The second change due to the introduction of contraception in the model is the need to reduce the post partum interval in order to simulate an accompanying reduction in breastfeeding. This is achieved by increasing the parameter, r, in the geometric distribution of delays from 1/6 to as much as 1/2 for highly contracepting populations. The means and variances of the post partum interval, including the one month fixed delay are shown in Table 4.4.

These two changes obviously have opposing effects on the level of fertility: the reduction in fecundability achieved through contraception reduces fertility whilst the reduction in the post partum interval serves to increase fertility.

Modifications of the Model

The Barrett simulation model had to be modified in order to produce age specific fertility schedules representative of early marrying, high fertility populations. This involved two important modifications. First, the model was extended to ages less than 20 with the accompanying introduction of a variable age at marriage; and secondly, marital dissolution was introduced to allow age specific fertility rates for all women to be calculated. Such modifications take no account of possible correlations between, for instance, early marriage and high fecundability. Since only changes in fertility patterns are of interest, however, this is not of importance.

Age at marriage

The previously fixed age at marriage parameter

Table 4.4: Mean lengths and variances of post partum intervals in lunar months determined by r

r	.1667	.2	.25	.3	.35	.4	.5
mean	11	9	7	5.7	4.7	4	3
variance	60	40	24	15.6	10.6	7.5	4

was replaced by the same function of proportions ever married, G(x), as was used in the Coale-Trussell fertility model described in Chapter 3. Again, values were obtained from the analytical expression developed by McNeil (Coale and McNeil, 1972)

$$g(x) = \frac{0.19465}{k} \exp\{\frac{-0.174}{k}(x-a_o-6.06k) - \exp[\frac{-0.2881}{k}(x-a_o-6.06k)]\}$$

where g(x) is first marriage frequency and a_o and k relate to the start and pace of first marriage respectively. The earliest age of start of first marriage allowed in the modified model is 10 years or 130 lunar months. The choice of a random number between 0 and 1 determines the point on the cumulative distribution, which in turn determines age at marriage in months. No account is made of the proportion who never marry, though the G(x) function does allow some women (depending on a_o and k) to marry at ages beyond the end of childbearing. The level of fertility obtained is therefore close to, but not exactly, the completed fertility rate for all women. This is not of importance for present purposes, since only the pattern of fertility is of interest. The cumulative function, proportions ever married, is reproduced by lunar month for k = 1.0 in Table 4.5. The effect of changing k can be seen from this table. For example, if k = 0.6, marriage occurs at a rate that is 1.0 ÷ 0.6 = 1.67 times as fast as when k = 1.0. Hence, after n months of marriage with k = 0.6, the same proportion

Table 4.5: Proportions ever married by year and lunar month

					LUNAR MONTH WITHIN YEAR		
1	2	3	4	5	6	7	8
.00016	.00033	.00052	.00076	.00098	.00124	.00151	.00185
.00447	.00503	.00567	.00634	.00706	.00784	.00867	.00955
.01622	.01758	.01901	.02052	.02211	.02378	.02553	.02737
[illegible]	.04270	.04520	.04791	.05066	.05350	.05644	.05947
.07965	.08334	.08713	.09100	.09497	.09902	.10316	.10739
[illegible]	.13926	.14412	.14904	.15404	.15911	.16424	.16943
[illegible]	.20792	.21293	.21854	.22428	.23001	.23577	.24157
.27593	.28204	.28849	.29487	.30108	.30490	.31142	.31595
.35515	.36117	.36719	.37318	.37917	.38514	.39110	.39705
.43212	.43712	.44314	.44904	.45510	.46105	.46672	.47235
.50507	.51057	.51623	.52156	.52695	.53211	.53732	.54250
.57275	.57755	.58251	.58733	.59211	.59694	.60154	.60614
[illegible]	.63701	.64143	.64521	.64955	.65404	.65840	.66291
.68716	.69075	.69435	.69810	.70181	.70548	.70911	.71270
.73343	.73675	.74006	.74327	.74650	.74967	.75281	.75592
.77340	.77654	.77949	.78229	.78504	.78777	.79047	.79313
[illegible]	.81092	.81333	.81572	.81808	.82041	.82271	.82499
[illegible]	.84014	.84223	.84426	.84625	.84823	.85017	.85212
.86323	[illegible]	.86673	.86843	.87014	.87186	.87352	.87519
[illegible]	.88617	.88755	.88902	.89046	.89188	.89329	.89467
.90253	.90340	.90513	.90638	.90730	.90861	.90989	.91116
.91749	.91845	.92001	.92105	.92219	.92309	.92409	.92509
[illegible]	.93195	.93256	[illegible]	.93428	.93514	.93598	.93681
.94179	.94236	.94310	.94363	.94455	.94528	.94600	.94670
[illegible]	.95135	.95144	.95201	.95322	.95403	.95483	.95552
[illegible]	[illegible]	.95697	.95949	.96031	.96102	.96152	.96202
[illegible]	.96341	.96376	.96420	.96584	.96707	.96749	.96791
.97040	[illegible]	.97105	.97142	.97179	.97215	.97251	.97286
[illegible]	[illegible]	.97551	.97582	.97612	.97643	.97673	.97702
.97871	[illegible]	.97925	.97951	.97977	.98002	.98027	.98052
.98194	.98217	.98234	.98261	.98283	.98304	.98326	.98345
[illegible]	.98475	.98508	.98522	.98540	.98558	.98576	.98594
[illegible]	[illegible]	.98720	.98741	.98757	.98772	.98787	.98802
[illegible]	[illegible]	.98913	.98925	.98939	.98951	.98964	.98976
[illegible]	.99054	.99070	.99081	.99091	.99102	.99113	.99123
.99193	[illegible]	.99201	.99211	.99220	.99229	.99238	.99246
[illegible]	.99306	.99312	.99320	.99326	.99335	.99343	.99350
.99397	[illegible]	.99405	.99412	.99414	.99425	.99431	.99437
.99473	.99475	.99486	.99489	.99495	.99500	.99505	.99510
.99530	[illegible]	.99549	.99556	.99559	.99563	.99567	.99572
.99597	.99601	.99605	.99609	.99612	.99616	.99620	.99624
.99645	.99647	.99651	.99656	.99659	.99661	.99664	.99667
.99685	.99687	.99690	.99693	.99696	.99698	.99701	.99703
.99718	.99721	.99723	.99725	.99728	.99730	.99732	.99734
.99747	.99749	.99751	.99752	.99754	.99756	.99758	.99760
.99770	.99772	.99774	.99775	.99777	.99778	.99780	.99782
.99790	.99792	.99793	.99795	.99796	.99797	.99798	.99800
.99807	.99808	.99809	.99811	.99812	.99813	.99814	.99815
.99821	.99822	.99823	.99824	.99825	.99826	.99827	.99828
.99833	.99834	.99835	.99836	.99836	.99837	.99838	.99839

after start of first marriage for k=1.0

9	10	11	12	13	YEARS AFTER MARRIAGE
.00219	.00254	.00299	.00344	.00394	1
.01050	.01151	.01259	.01373	.01494	2
.02929	.03130	.03340	.03559	.03787	3
.06260	.06582	.06914	.07255	.07605	4
.11170	.11609	.12057	.12513	.12976	5
.17463	.17999	.18535	.19077	.19624	6
.24740	.25326	.25914	.26505	.27098	7
.32499	.33102	.33706	.34309	.34912	8
.40298	.40889	.41478	.42065	.42650	9
.47745	.48352	.48956	.49555	.50003	10
.54704	.55274	.55780	.56282	.56781	11
.61081	.61438	.61791	.62140	.62484	12
.66699	.67102	.67501	.67896	.68287	13
.71425	.71876	.72323	.72667	.73007	14
.75898	.76202	.76501	.76798	.77091	15
.79577	.79837	.80094	.80348	.80598	16
.82723	.82945	.83164	.83381	.83595	17
.85403	.85592	.85778	.85962	.86144	18
.87678	.87838	.87996	.88152	.88306	19
.89604	.89740	.89873	.90005	.90135	20
.91232	.91346	.91459	.91570	.91680	21
.92606	.92702	.92797	.92891	.92983	22
.93784	.93865	.93945	.94004	.94062	23
.94739	.94807	.94875	.94941	.95007	24
.95560	.95617	.95674	.95730	.95785	25
.96251	.96299	.96347	.96394	.96440	26
.96832	.96872	.96913	.96952	.96991	27
.97320	.97355	.97388	.97422	.97455	28
.97731	.97760	.97788	.97816	.97844	29
.98077	.98101	.98125	.98148	.98171	30
.98367	.98387	.98407	.98427	.98447	31
.98611	.98628	.98645	.98662	.98678	32
.98816	.98830	.98845	.98859	.98872	33
.98990	.99001	.99012	.99024	.99036	34
.99133	.99143	.99153	.99163	.99173	35
.99255	.99264	.99272	.99280	.99288	36
.99357	.99364	.99371	.99378	.99385	37
.99443	.99449	.99455	.99461	.99467	38
.99515	.99521	.99525	.99530	.99535	39
.99576	.99580	.99585	.99589	.99593	40
.99627	.99631	.99634	.99638	.99641	41
.99670	.99673	.99676	.99679	.99682	42
.99706	.99709	.99711	.99713	.99716	43
.99736	.99738	.99740	.99743	.99745	44
.99762	.99763	.99765	.99767	.99769	45
.99783	.99785	.99786	.99788	.99789	46
.99801	.99802	.99804	.99805	.99806	47
.99816	.99817	.99818	.99819	.99820	48
.99829	.99830	.99831	.99831	.99832	49
.99839	.99840	.99841	.99842	.99842	50

have married as after 0.6 n months with k = 1.0. This assumes the same value of a_0. Obviously, changes in a_0 have the effect of moving the curve along the age axis, so that as a_0 increases, marriage is generally later.

Teenage fecundability

The introduction of early ages at marriage into the model means that age at menarche and teenage sub-fertility need to be taken into account. Age at menarche is not dealt with directly; rather, fecundability is reduced at ages less than 20 by an exponential function chosen to account for both factors. The development of this function is given in Appendix 4.1, the end result being a fecundability at ages less than 20 equal to

$$\rho = \rho^* \exp\{(x - 260)/40\}$$

where ρ^* is the basic monthly fecundability determined randomly from the beta distribution given in equation 4.1. The values of ρ at ages 10 to 19 for $\rho^* = 100$ are given in Table 4.6.

Outcome of teenage pregnancy

The age dependent functions of the probabilities of foetal death, stillbirth and livebirth (θ_1, θ_2 and θ_3)

Table 4.6: Fecundability at ages 10 to 19 when $\rho^* = 100$

Age		
Years	Months	Fecundability
10	130	4
11	143	5
12	156	7
13	169	10
14	182	14
15	195	20
16	208	27
17	221	38
18	234	52
19	247	72
20	260	100

are not suitable for ages less than 20 years. These functions imply that the probability of a livebirth increases linearly for younger and younger ages; clearly this is not the case.

In the absence of any clearcut evidence, and to keep the model from becoming too cumbersome, values of θ_1, θ_2 and θ_3 are assumed to be the same at ages less than 20, as at 20 years. Hence, for teenage pregnancies, the probability of foetal death is 0.19, of stillbirth 0.02 and of livebirth 0.79. At very young ages, where these probabilities may be erroneous (in favour of livebirth), the small number of pregnancies involved means that the effect on fertility is minimal.

Modified sterility

In its unmodified form, the Barrett simulation model produces marital fertility schedules, and has no provision for the effects on fertility of marital dissolution. In order to determine age specific fertility for all women, it was therefore necessary to allow for the effects of widowhood, separation and divorce on fertility. This was achieved by changing the sterility function to include the effects of marital dissolution. The age at menopause function was left unchanged.

The original age at sterility function is

$$x_s = 28 + z/0.012$$

where x_s is age at sterility and z is a random variable between 0 and 1. After a series of modifications, described in Appendix 4.2, this was replaced by an exponential function:

$$x_s = 20 + 13 \ln(1 + 9z)$$

As before, the younger of the ages of sterility (where 'sterility' now includes marital dissolution) and menopause is taken as marking the end of the possible childbearing period. The two distributions are illustrated in Figure 4.1. The introduction of the possibility of early ages at sterility requires that the fecundability function also be modified slightly. Where the age at the end of the possible childbearing period, x_e, is greater than 30, ρ decreases monotonically from ρ^* at 30 to 0 at x_e (for contracepting populations, the factor $1 - E_i$ is applied) as before. Where x_e is less than or equal to 30 years, no such decline occurs, and ρ becomes zero immediately. This is equivalent to stating that all women who become 'sterile' before the age of 30, do so because of marital dissolution rather than biological factors.

Contraceptive effectiveness level

The above modifications were all made using contraceptive effectiveness of $E_1 = 0.7$ and $E_2 = 0.9$. This was to avoid the very high fertility characteristic of populations such as the Hutterites. The actual effect of this low level of contraception is small, however.

Simulating a Fertility Decline

Taking the modified Barrett simulation model as described above, a gradual decline in fertility was achieved by changes in parameters governing the level and pattern of fertility. The level of completed fertility produced by the modified model is roughly 8.0 for women who ever marry, for early marrying populations with nuptiality parameters a_o of about 10 or 11 years and k of about 0.5. (Note that all completed fertility levels produced by the simulation are for ever married women and are therefore slightly higher than the level for all women.) The parameters used to bring about a decline in fertility are desired family size, contraceptive effectiveness in combination with length of post partum interval, and the nuptiality parameters a_o and k. Their separate effects are discussed below, before considering their combined effect as a declining fertility situation. In all cases, results should be viewed in the light of sampling errors, discussed in Appendix 4.7.

Desired family size parameter

In all previous simulation runs, the desired family size parameter, DFS, was set equal to an unachievable quantity (40), so that it had no effect on fertility. Introduction of the variable at lower levels produced a lower level of fertility as well as a younger and more peaked pattern. Results of decreasing DFS for a_0 = 130 lunar months, k = 0.6, and for contraceptive effectiveness, E_1 and E_2 of 0.7 and 0.9 respectively coupled with a post partum coefficient, r, of 1/6 are shown in Table 4.7. As expected, this parameter is very important in producing the desired fertility decline.

Consideration was given to whether or not DFS should be a variable parameter rather than fixed for all women. Various simulation exercises were carried out with weighted combinations of desired family sizes. Comparison with single values of DFS showed that for the range of values tested, variability of DFS has very little effect on fertility achievement. The parameter was thus left as a fixed quantity. A more detailed account of these comparisons is given in Appendix 4.3.

Post partum interval and contraceptive effectiveness parameters

In previous runs the post partum coefficient was set at r = 1/6, equivalent to an interval length of 11 lunar months. Changes in this parameter are only meaningful if coupled with an increase in contraceptive effectiveness, since short post partum intervals are associated with more

Table 4.7: Effect of reducing desired family size on the level and pattern of fertility

	Desired family size			
Age	(40)	6	5	4
10-14	.00533	.00713	.00984	.01283
15-19	.11034	.15011	.16721	.18759
20-24	.24327	.31640	.34311	.34460
25-29	.23807	.26961	.25092	.23337
30-34	.19464	.15829	.13886	.12984
35-39	.13205	.06853	.06075	.06246
40-44	.06145	.02592	.02430	.02332
45-49	.01460	.00365	.00501	.00578
50-54	.00025	.00035	-	.00021
level	7.88	5.75	5.19	4.68
%	100	73	66	59
%	137	100	90	81

$a_o = 130, k = 0.6, E_1 = 0.7, E_2 = 0.9, r = 1/6$

developed societies which are also likely to be highly contracepting. The effect on fertility of changes in r alone are shown here to identify their contribution to the combined effect. Table 4.8a gives results for $a_0 = 130$, $k = 0.6$, DFS = 5, and the low contraceptive effectiveness $E_1 = 0.7$ and $E_2 = 0.9$. In fact, reducing the length of the post partum interval from 11 to 5.7 months has very little effect on the pattern of fertility (in the direction of small reductions in the mean and variance), but does increase the level slightly. This increase in level is much reduced, however, when $E_1 = 0.9$ and $E_2 = 0.99$ as shown in Table 4.8b.

The effect of increasing contraceptive effectiveness from the low level of $E_1 = 0.7$, $E_2 = 0.9$ to the high level of $E_1 = 0.9$, $E_2 = 0.99$ is seen by comparing the columns of Tables 4.8a and 4.8b. The reduction of completed fertility is greater for shorter post partum intervals, as shown by the column percentages in the bottom row of the table. The effect on the pattern of fertility, also more pronounced for shorter post partum intervals, is to move the curve slightly to younger ages and to make the curve more peaked, that is both the mean and variance are reduced.

The effects of reducing the post partum interval and increasing contraceptive effectiveness are in the same direction for the pattern of fertility, namely of reducing both the mean and variance. The combined effect

Table 4.8: Effect of reducing the post partum interval on the level and pattern of fertility for low and high contraceptive effectiveness

Age	Value of r and interval in lunar months			
	1/6	.2	.25	.3
	11	9	7	5.7
a) low contraceptive effectiveness ($E_1 = 0.7$, $E_2 = 0.9$)				
10-14	.00984	.01103	.01263	.00869
15-19	.16721	.16601	.16874	.18564
20-24	.34311	.35315	.35578	.35975
25-29	.25092	.25014	.24744	.23706
30-34	.13886	.13591	.12775	.13174
35-39	.06075	.05496	.06076	.05160
40-44	.02430	.02356	.02105	.02199
45-49	.00501	.00486	.00549	.00337
50-54	-	.00037	.00037	.00018
level	5.19	5.35	5.46	5.64
%	100	103	105	109
b) high contraceptive effectiveness ($E_1 = 0.9$, $E_2 = 0.99$)				
10-14	.01138	.00870	.01131	.01146
15-19	.18795	.20339	.19487	.20649
20-24	.39330	.39407	.41409	.42357
25-29	.25558	.25446	.24989	.23611
30-34	.11027	.09768	.09004	.08649
35-39	.03125	.03256	.02719	.02422
40-44	.00848	.00781	.00826	.00865
45-49	.00156	.00134	.00391	.00303
50-54	.00022	-	.00043	-
level	4.48	4.48	4.60	4.63
%	100	100	103	103
column %	86	84	84	82

$a_o = 130$, $k = 0.6$, DFS = 5

can be seen by comparing the first column of Table 4.8a with the last column of Table 4.8b. For the level, however, their effects are in opposite directions. The reduction in completed fertility due to increased contraceptive effectiveness is greater than the increase due to the shorter post partum interval, so that the combined effect is to reduce completed fertility to 89 per cent.

Nuptiality parameters

Fertility declines in developing societies are usually accompanied by rises in the age at marriage and a slowing down in the pace at which marriage occurs. Such changes obviously have a delaying effect on fertility resulting in reduced fertility because women are less fecund at older ages.

The effects of increasing a_0, the start of first marriage, from 10 years to 15 years for a fixed value of $k = 0.5$ are shown in Table 4.9 (for DFS = 6, $r = 1/6$, $E_1 = 0.7$ and $E_2 = 0.9$). The effect on the level of fertility is small for very young ages where fecundability is low but increases by increasing amounts as higher ages are reached. The age pattern of fertility is compressed slightly by virtue of the fact that the reproductive span is shortened.

The effect of increasing k, that is slowing down

Table 4.9: Effect of increasing age of start of first marriage on the level and pattern of fertility for k = 0.5

	Exact age of start of first marriage (years)					
Age	10	11	12	13	14	15
10-14	.01133	.00545	.00259	.00036	-	-
15-19	.16754	.15072	.13702	.10195	.07160	.04641
20-24	.31784	.32340	.31868	.32114	.31136	.28366
25-29	.25782	.26601	.28576	.29982	.30457	.31569
30-34	.15199	.15497	.16132	.17034	.19185	.21366
35-39	.06340	.06795	.06756	.07584	.08665	.10261
40-44	.02553	.02520	.02051	.02575	.02846	.03280
45-49	.00423	.00613	.00603	.00444	.00496	.00518
50-54	.00034	.00017	.00052	.00036	.00055	-
level	5.92	5.87	5.80	5.63	5.45	5.21
%	100	99	98	95	92	88

DFS = 6, r = 1/6, E_1 = 0.7, E_2 = 0.9

the pace of marriage, can be seen in Table 4.10 (again for DFS = 6, r = 1/6, E_1 = 0.7, E_2 = 0.9 and a_o = 10 years). The resulting decrease in completed fertility is greater than that resulting from increasing the age at start of marriage. The effect on the pattern of fertility is very similar to the effect of increasing a_o from 10 to 15 years. The combined effect of a_o and k is shown in Table 4.11 (for DFS = 6, r = 1/6, E_1 = 0.7, E_2 = 0.9) using the same combinations that are used later in the actual simulation of a fertility decline. As expected, the decrease in total fertility is greater than for either separate effect, but does not appear to be as great as the sum of the two effects (though sampling errors could account for this). The effect of increasing both parameters simultaneously on the pattern of fertility is very similar to the separate effects though slightly more pronounced.

Relative effect of parameters

It is seen from the above analysis that the parameter that has most effect on the level of fertility is desired family size. A reduction from DFS = 6 to DFS = 4 results in a 19% fall in completed fertility, a decline which is only equalled by the combined effect of changing nuptiality parameters from a_o = 10 years, k = 0.6 to a_o = 15 years, k = 1.0. The effect of k the pace of marriage, can be seen in Table 4.10 (again for DFS = 6, r = 1/6, E_1 = 0.7, E_2 = 0.9 and a_o = 10 years). The resulting decrease in completed fertility is greater than that resulting from increasing the age at start of marriage. The effect on the pattern of fertility is very similar to the effect of increasing a_o from 10 to 15 years. The combined effect of a_o and k is shown in Table 4.11 (for DFS = 6, r = 1/6, E_1 = 0.7, E_2 = 0.9) using the same combinations that are used later in the actual simulation of a fertility decline. As expected, the decrease in total fertility is greater than for either separate effect, but does not appear to be as great as the sum of the two effects (though sampling errors could account for this). The effect of increasing both parameters simultaneously on the pattern of fertility is very similar to the separate effects though slightly more pronounced.

Relative effect of parameters

It is seen from the above analysis that the parameter that has most effect on the level of fertility is desired family size. A reduction from DFS = 6 to DFS = 4 results in a 19% fall in completed fertility, a decline which is only equalled by the combined effect of changing nuptiality parameters from a_o = 10 years, k = 0.6 to a_o = 15 years, k = 1.0. The effect of k

Table 4.10: Effect of decreasing the pace of first marriage (increasing k) on the level and pattern of fertility for a_0 = 10 years

Age	Value of k					
	0.5	0.6	0.7	0.8	0.9	1.0
10-14	.01133	.00713	.00568	.00700	.00397	.00222
15-19	.16754	.15011	.13317	.11670	.10004	.08461
20-24	.31784	.31640	.31055	.29431	.27439	.26128
25-29	.25782	.26961	.27699	.28371	.29160	.29311
30-34	.15199	.15829	.17330	.18082	.19535	.21253
35-39	.06340	.06853	.06836	.08152	.09777	.10556
40-44	.02553	.02592	.02628	.02988	.03026	.03425
45-49	.00423	.00365	.00550	.00605	.00662	.00624
50-54	.00034	.00035	.00018	-	-	.00020
level	5.92	5.75	5.63	5.29	5.29	4.96
%	100	97	95	89	89	84

DFS = 6, r = 1/6, E_1 = 0.7, E_2 = 0.9

Table 4.11: The combined effect of increasing age at start of first marriage and decreasing pace of first marriage on the level and pattern of fertility

Age	Parameters* (a_o, k)				
	(130, 0.6)	(136, 0.7)	(143, 0.8)	(149, 0.9)	(156, 1.0)
10-14	.00713	.00521	.00344	.00102	.00022
15-19	.15011	.11921	.09981	.07258	.05169
20-24	.31640	.30467	.28203	.25516	.23928
25-29	.26961	.28205	.29885	.29708	.29442
30-34	.15829	.17271	.19522	.21427	.23239
35-39	.06853	.08330	.08623	.11000	.12190
40-44	.02592	.02675	.02945	.04069	.05061
45-49	.00365	.00592	.00497	.00920	.00926
50-54	.00035	.00018	-	-	.00022
level	5.75	5.57	5.23	4.89	4.64
%	100	97	91	85	81

* a_o is measured in lunar months

DFS = 6, r = 1/6, E_1 = 0.7, E_2 = 0.9

alone is the second most important factor in bringing about a fertility decline. The other parameters, contraceptive effectiveness and post partum interval, have less overall effect because of their opposite separate effects on level.

The pattern of fertility is affected considerably by the various parameters. The greatest effect on the mean is produced by the nuptiality paramaters, a_0 and k, increases in which move the fertility curve towards older ages, without having much effect on the variance. Reducing the desired family size parameters has considerable effect in reducing the variance and also reduces the mean somewhat. The contraceptive effectiveness and post partum parameters both reduce the mean and variance slightly.

Combined effect of parameters on fertility: declining fertility

A declining fertility situation was simulated in 5 stages. The parameters discussed above were changed in combination and their values at each stage are shown in Table 4.12. The amount of change in the parameters is roughly equal over the five stages though not necessarily over time: no attempt is made here to put a time span on the simulated decline. The a_0 parameter does not reach values higher than 12 years to allow for some fertility to occur during the first age group, 10-14 years. The delay in marriage is rather accounted for by the parameter k: at stage 1, 83% of women who ever marry have done so

Table 4.12: Values of parameters contributing to declining fertility at each stage of the decline

Stage	a_o*	k	DFS	r	E_1	E_2
1	130	.6	6	1/6	.7	.9
2	136	.7	6	.2	.75	.923
3	143	.8	5	.25	.8	.945
4	149	.9	5	.3	.85	.968
5	156	1.0	4	.35	.9	.99

*a_o is measured in lunar months

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4	149	.9	5	.3	.85	.968
5	156	1.0	4	.35	.9	.99

*a_o is measured in lunar months

by age 20; at stage 5 this proportion is 35%. The desired family size parameter is reduced slowly, because of its considerable effect on both the level and pattern of fertility, to a lower limit of 4 (Caldwell, 1974). Contraceptive effectiveness ranged from the low level used throughout the development of the modified model to a high level of $E_1 = 0.9$, $E_2 = 0.99$. This seemingly highly effective (99%) level does, in fact, allow for some "mistaken" conceptions to occur, though these are more than outweighed, in terms of completed fertility, by the failure of some women to achieve the desired family size. The post partum interval ranges from 11 months ($r = 1/6$) to a reasonably developed society value of 4.7 months ($r = .35$).

The age specific fertility rates (normalised to sum to 1) and completed fertility for each stage of the fertility decline are shown in Table 4.13. Completed fertility is reduced by 44% over the 5 stages, and the pattern changes from a high early peak to a less peaked later distribution.

Comparisons with Knodel's work

Knodel (1977) compares the age patterns of fertility of contemporary Asia and pre-industrial Europe by consideration of their values of m, the index of voluntary birth control in the Coale-Trussell model.

Table 4.13: Age specific fertility rates at successive stages of the simulated fertility decline

Age	Stage of fertility decline				
	1	2	3	4	5
10-14	.00713	.00649	.00216	.00257	.00123
15-19	.15011	.12232	.10953	.09002	.07683
20-24	.31640	.32463	.35403	.33037	.31780
25-29	.26961	.28662	.29474	.30302	.31996
30-34	.15829	.16303	.15287	.17863	.18143
35-39	.06853	.06990	.06080	.06617	.07436
40-44	.02592	.02198	.02285	.02455	.02345
45-49	.00365	.00486	.00259	.00444	.00463
50-54	.00035	.00018	.00043	.00023	.00031
level	5.75	5.55	4.64	4.28	3.24
%	100	97	81	74	56
Age	Ratios of rates to 20-24 rate				
10-14	.02	.02	.01	.01	.00
15-19	.47	.38	.31	.27	.24
20-24	1.00	1.00	1.00	1.00	1.00
25-29	.85	.88	.83	.92	1.01
30-34	.50	.50	.43	.54	.57
35-39	.22	.22	.17	.20	.23
40-44	.08	.07	.06	.07	.07
45-49	.01	.01	.01	.01	.01
50-54	.00	.00	.00	.00	.00

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level	5.75	5.55	4.64	4.28	3.24
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Age	Ratios of rates to 20-24 rate				
10-14	.02	.02	.01	.01	.00
15-19	.47	.38	.31	.27	.24
20-24	1.00	1.00	1.00	1.00	1.00
25-29	.85	.88	.83	.92	1.01
30-34	.50	.50	.43	.54	.57
35-39	.22	.22	.17	.20	.23
40-44	.08	.07	.06	.07	.07
45-49	.01	.01	.01	.01	.01
50-54	.00	.00	.00	.00	.00

A similar comparison, between the simulated fertility schedules and Knodel's Asian data, is carried out here to ascertain the validity of the simulated age pattern of fertility.

A description of the Coale-Trussell model of fertility has already been given in Chapter 3, and the method used by Knodel of calculating m from age-specific marital fertility rates appears in Appendix 3.1B. In order to be able to calculate m values for the simulated data, however, the rates (which are for ever-married women) need to be converted into rates for currently married women only. This was attained to a limited degree of satisfaction (see Appendix 4.4) by re-adopting the original sterility function of the Barrett simulation model. The m values resulting from this appear in Appendix Table A4.4.1. The standard deviations of the m values in each schedule are well within the levels found by Knodel, indicating that the age pattern of fertility is consistent with both empirical evidence and the Coale-Trussell model.

Knodel also compares the age patterns of fertility decline of Asia and pre-industrial Europe, by calculating the percentage changes in marital fertility during different stages of fertility transition. A similar comparison between the simulated fertility decline and Knodel's Asian data is made in Appendix 4.5, using the simulated marital fertility rates obtained in

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Appendix 4.4 by using Barrett's sterility function. The pattern of percentage changes found in the simulated data is well in line with Knodel's findings over periods where the desired family size parameter has changed. Where DFS has not changed, however, there is no clear pattern of change. It is suggested that periods that incorporate a change in DFS might be of more interest than those that do not.

Comparisons with Lesthaeghe's work

The effect of nuptiality patterns on fertility in general have been well documented (Coale and Tye, 1961; Leasure, 1963; Coale, 1967, 1971; Talwar, 1967, 1974). Lesthaeghe (1971), however, looks more closely at the patterns of nuptiality and the effect of nuptiality changes on marital fertility. His schedules of changing nuptiality are used as a reference with which the nuptiality schedules used in the simulated fertility decline are compared in Appendix 4.6. There are considerable differences in the parameters of the two transitions: the nuptiality parameters used in the simulation describe a much earlier start to marriage but a slower pace than do those used by Lesthaeghe. In addition, no account of possible changes in the final proportion ever married, C, is made in the simulation, whereas Lesthaeghe does incorporate a decline in C. Despite these obvious differences in the individual parameters, their combined effect as proportions ever married schedules are remarkably alike for the two transitions. The greatest discrepancies occur at very young and very old ages, though these differences are shown to be negligible in terms of their effect on age specific fertility.

CHAPTER 5

TESTING THE TRANSFORMED GOMPERTZ MODEL

Introduction

This chapter describes the procedure by which the transformed Gompertz model is fitted to data. The efficacy of the model is shown in its application to several sets of well-behaved data where completed fertility is known. In this chapter all analyses are performed using cumulative fertility rates, F(x), at exact ages 15, 20, etc. In the following chapter the model is adapted for use with data obtained from maternity histories. The basic fitting procedure described here is used throughout.

The Efficiency and Applicability of the Model

The empirical base of the Coale-Trussell model fertility schedules, which in turn form the basis of the standard fertility schedule used in the transformed Gompertz model, provides the model with much greater fitting powers than the ordinary Gompertz. This is shown in the non-linearity of $Y_s(x)$, especially in the tails of its distribution where the large deviations from linearity indicate that the Gompertz curve is rather a poor fit (see Chapter 2, Figure 2.1). The transformed Gompertz model is based on the more plausible assumption that fertility follows the pattern of the empirically based standard rather than of the ordinary Gompertz curve. The particular standard pattern of fertility developed in Chapter 3 is based on a subset of the Coale-Trussell model fertility schedules, namely those representative of high fertility populations. The use of this standard in the transformed Gompertz model should thus be restricted to the model's application to high fertility data. It is, of course, possible to develop other standard patterns of fertility from the Coale-Trussell set for use with lower fertility data. With good quality data, internal standards might be used.

In this chapter, the transformed Gompertz model is tested on data for which the high fertility standard is not strictly appropriate because of their lower level of fertility. Use of such data is unavoidable because good quality data with high fertility levels do not exist. The problem is not serious, however, because the results of the tests will be more conservative than if appropriate data were available. In general, therefore, the model might be expected to produce slightly better results than those reported here.

Fitting to Cumulative Fertility

The transformed Gompertz model developed in Chapter 2 is described by

$$F(x) = F\,P^{Q^{Y_s(x)}} \qquad 5.1$$

where F(x) is cumulative fertility to exact age x, $Y_s(x)$ is transformed standard fertility, and P, Q and F are parameters. F is of most interest being interpretable as completed fertility. The best fit in the least squares sense of the model to observed data is obtained by minimising the objective function

$$S = \sum_x w(x)\,[F(x) - \hat{F}(x)]^2 \qquad 5.2$$

where F(x) is observed, and $\hat{F}(x)$ is fitted, cumulative fertility and w(x) is a set of weights attached to F(x). The choice of w(x) = 1 and of the method of estimation (least squares) are those used in Chapter 6 for the analysis of data from maternity histories. The reasons for their choice are discussed in that chapter in relation to the nature of the data. Their use in the analysis of well-behaved data is not necessarily the best choice, but does provide a direct comparison with the results in Chapter 6.

The fitting procedure

The procedure used to fit the transformed Gompertz model to data is described here in terms of the minimisation of S in equation 5.2. The process is iterative and uses the general minimisation program MINUIT (James and Roos, 1971). All other computer programming is specific to this fitting procedure and appears in Appendix 5.1.

Initial estimates of the parameters, P, Q and F, required by MINUIT as a starting point, are derived from the data so as to provide a reasonably accurate start, thereby keeping the number of iterations to a minimum and avoiding possible local minima that may exist (in poor quality data) away from the region of the true minimum value of S.

The initial parameter estimates are calculated from estimates of the ordinary Gompertz parameters of both the standard and observed fertility as described in Appendix 5.2. This procedure is based on the assumption that the Gompertz fit is adequate for obtaining initial estimates for the transformed Gompertz model. The Gompertz parameters for the standard are derived in Appendix 3.3. Estimation of the ordinary Gompertz parameters for the observed data is done by the method of selected points (described in Appendix 5.3). Though this is a simple method of estimation with the possibility of large errors, especially for poor quality data, it is adequate for the purpose of providing initial estimates of P, Q and F.

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The Data and Results

The data discussed in this chapter are cohort fertility cumulated to exact ages 15, 20, ... 50, and are referred to as endpoint data. Such data have usually been collected by vital registration rather than by retrospective survey or census. Births are recorded by year of occurrence by age of mother so that it is possible to arrange the data to be cumulative to exact ages. By choosing to cumulate to exact ages 15, 20, ... 50, the data are appropriate for analysis by the transformed Gompertz model using the endpoint values of the standard, $Y_s(x)$, developed in Chapter 3. The data are presented in Appendix 5.4.

The results are presented in terms of F, α and β. The full results, including estimates of P and Q and a measure of goodness of fit based on the objective function, are presented in Appendix 5.5. The demographic interpretation of the parameters has been discussed in Chapter 2: briefly, P (where $\alpha = -\ln(-\ln P)$) represents the proportion of fertility achieved by the origin in the standard (approximately 24.9 years) and β ($= -\ln Q$) describes the speed at which fertility occurs relative to the standard. F is the level parameter measuring completed fertility.

Simulated Data

A series of high fertility cohort data incorporating a 'controlled' fertility decline were obtained by simulation. The object of this exercise was to produce a set of fertility rates free from reporting errors and biases. In addition, such simulated data are based on known parameters, so that changes in the pattern of fertility, in particular in the direction of declining fertility, can be brought about by known changes in parameters. The use of the transformed Gompertz model in a declining fertility situation can thus be examined.

The process by which the data were simulated is described in detail in Chapter 4. The simulation process produces birth histories for individual women, assigning each birth to the appropriate age group according to the exact age of mother. The data, when cumulated, thus refer to exact ages 15, 20, ... 50, that is to the endpoints of five-year age groups. Five schedules of cumulative cohort fertility were simulated representing five consecutive stages of a fertility decline. Since only the changing pattern of fertility is of interest, the rates are normalised to sum to unity. However, the simulation model allows for births to occur to age 54, whilst the transformed Gompertz model assumes zero fertility after exact age 50. Actual fertility at 50 thus falls slightly short of 1.

Cumulative fertility rates for the five stages of the simulated fertility decline are reproduced in Appendix 5.4 (Table A5.4.1). The estimates of completed fertility obtained by fitting the transformed Gompertz model appear in Table 5.1. For each stage of the decline the model was fitted to all eight datapoints (at ages 15, 20, ..., 50) to provide an 'overall' estimate of F (in all cases less than 0.5 per cent in error). As expected, reducing the number of datapoints to which the model is fitted, as if the data referred to incomplete cohort experience, results in poorer estimates of F. This is seen in the lower half of Table 5.1 where percentage errors appear. Fitting to relatively complete data, that is including up to at least age 40, results in slight underestimates of F, though not by more than 1.0 per cent. The inclusion of progressively fewer points causes this underestimate to become an overestimate. For these data, fitting to age 30, that is to only four datapoints, results in an error of at most 6.20 per cent. The early part of the data thus points towards slightly higher completed fertility than is observed, but as later datapoints are included expectations change towards a very slight underestimate of the final level.

Examination of the estimates of the parameters governing the shape, rather than the level, of fertility provides a clearer understanding of the model. Estimates of α and β are shown in Table 5.2. It is seen that within stages of the fertility decline an increase in F is generally associated with decreases in both α and β, and vice versa. There are exceptions to this,

Table 5.1: Estimates of completed fertility for simulated data

Points included	Stage of fertility decline				
	1	2	3	4	5
15 to 50	.99679	.99686	.99698	.99552	.99548
15 to 45	.99760	.99675	.99725	.99533	.99583
15 to 40	.99775	.99751	.99050	.99291	.99325
15 to 35	1.01712	1.01233	.99614	1.01002	1.00474
15 to 30	1.05201	1.06186	1.02366	1.02535	1.05248
Actual	.99964	.99983	.99957	.99977	.99969
	Percent error in estimate				
15 to 50	-.29	-.30	-.26	-.43	-.42
15 to 45	-.20	-.31	-.23	-.44	-.39
15 to 40	-.19	-.23	-.91	-.69	-.64
15 to 35	1.75	1.25	-.34	1.03	.51
15 to 30	5.24	6.20	2.41	2.56	5.28

Table 5.2: Estimates of α and β for simulated data

Points included	Stage of fertility decline				
	1	2	3	4	5
			α estimates		
15 to 50	.28378	.22077	.24249	.12801	.05924
15 to 45	.28200	.22322	.24276	.12823	.05873
15 to 40	.28169	.22154	.25429	.13191	.06191
15 to 35	.24821	.19660	.24452	.10643	.04632
15 to 30	.19555	.13012	.20217	.08631	-.01031
			β estimates		
15 to 50	1.31914	1.38417	1.49064	1.45942	1.47500
15 to 45	1.32016	1.38737	1.48850	1.46025	1.47755
15 to 40	1.32001	1.38422	1.51476	1.446917	1.48429
15 to 35	1.27265	1.34651	1.49857	1.42130	1.45219
15 to 30	1.22308	1.26846	1.44357	1.39498	1.37364

in that a change in the estimate of F may be associated with changes in α and β of opposite directions, but in such cases the dominant change is in the opposite direction to the change in F. Higher estimates of F are thus associated with lower estimates of α and/or β, and vice versa. In the case of α, this association is readily understood: lower values of α indicate that a smaller proportion of observed fertility is achieved by age x_{os}. Since the actual values of observed fertility remain fixed, a lower (higher) estimate of α (for given β) can only be achieved by increasing (decreasing) the estimate of F. In the case of β, the association between the pace and level of fertility may not be obvious. However, low (high) values of β (for given α) indicate that the distribution of fertility by age has a relatively large (small) variance, that is that the rates at very young and very old ages are relatively high (low). Within stages of the fertility decline, therefore, lower (higher) estimates of β indicate that the fitted model assumes higher (lower) rates of fertility at the final stages of childbearing such that F is also higher (lower). Hence, higher estimates of the level of fertility are associated with a generally later and slower pattern, and lower estimates of the level are associated with generally earlier and faster pattern.

Trends in α and β

The simulated data allow for the changes in the pattern of fertility that occur as the level declines to be examined in detail because of the known generating parameters. The effects of these parameters on fertility are discussed fully in Chapter 4, and an attempt is made here to relate them to α and β. This is done for the estimates of α and β obtained by fitting to the complete data, but it should be noted that the same relationships hold for α and β estimates from incomplete data.

The parameter values and α and β estimates for each stage of the simulated fertility decline are reproduced in Table 5.3. It is seen that, apart from an irregularity at stage 3, α decreases and β increases as the fertility decline progresses. In other words, a lower level of fertility is associated with a later and more peaked pattern. This is entirely consistent with the values of the simulation generating parameters. The steady increase in a_0, age at start of first marriage, and the accompanying decrease in the rate at which marriage occurs (increasing k) are the dominant changes affecting the mean age of fertility, the increasing value of which is clearly reflected in the decreasing estimates of α. The reduction in the desired family size parameter is the main cause of the decreases in the variance as

Table 5.3: Generating parameters and α and β estimates for the stages of the simulated fertility decline

	Stage of fertility decline				
	1	2	3	4	5
a_o*	130	136	143	149	156
k	.6	.7	.8	.9	1.0
DFS	6	6	5	5	4
r	1/6	.2	.25	.3	.35
E_1	.7	.75	.8	.85	.9
E_2	.9	.923	.945	.968	.99
α	.28	.22	.24	.13	.06
β	1.32	1.38	1.49	1.46	1.48

* measured in lunar months

the fertility decline progresses, helped by increasing contraceptive effectiveness and hence progressively successful achievement of desired family size. This is reflected in the increasing values of β.

The irregularity in the trends in α and β at stage 3 can be related to the changes in the generating parameters at that stage. The increase, rather than decrease, in α and the large increase in β are brought about a combination of factors. The desired family size parameter is of necessity reduced by integer values, so that at some stages it was necessary to retain the same value to avoid too great a decline in the level of fertility. The reduction of this parameter from 6 to 5 at stages 2 to 3, coupled with the effect of large coefficients of variation for age specific rates at very young and very old ages (see Appendix 4.7) produced a stage 3 schedule with very low rates at 10-14 and 45-49. The effect is to reduce the variance considerably, and also to reduce late fertility sufficiently to push the mean towards younger ages. To describe this effect on α and β as an irregularity is not to say that it would be unlikely to occur in real data. Indeed, such an effect would be expected in populations where family planning programmes have a sizeable impact on the fertility of older women, but where marriage and early fertility patterns are changing only slowly.

It might be expected that the reduction of the fertility decline progresses, helped by increasing contraceptive effectiveness and hence progressively successful achievement of desired family size. This is reflected in the increasing values of β.

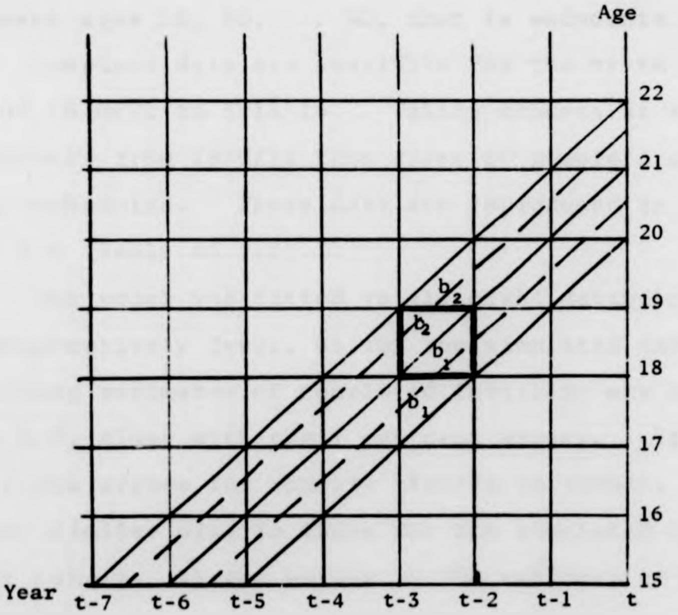
The irregularity in the trends in α and β at stage 3 can be related to the changes in the generating parameters at that stage. The increase, rather than decrease, in α and the large increase in β are brought about a combination of factors. The desired family size parameter is of necessity reduced by integer values, so that at some stages it was necessary to retain the same value to avoid too great a decline in the level of fertility. The reduction of this parameter from 6 to 5 at stages 2 to 3, coupled with the effect of large coefficients of variation for age specific rates at very young and very old ages (see Appendix 4.7) produced a stage 3 schedule with very low rates at 10-14 and 45-49. The effect is to reduce the variance considerably, and also to reduce late fertility sufficiently to push the mean towards younger ages. To describe this effect on α and β as an irregularity is not to say that it would be unlikely to occur in real data. Indeed, such an effect would be expected in populations where family planning programmes have a sizeable impact on the fertility of older women, but where marriage and early fertility patterns are changing only slowly.

It might be expected that the reduction of desired family size from 5 to 4 at stages 4 to 5 would produce a similar effect. It seems, however, that at these later stages, increased contraceptive effectiveness and later marriage are more important determinants of the pattern of fertility than a reduction in desired family size from 5 to 4. (This would not be so if desired family size were reduced from 2 to 1.) Thus α continues to decrease, and the increase in β is relatively small.

Swedish Data

The Swedish data are produced by the National Central Bureau of Statistics (Statistika centralbyran, 1969) and are based on the registration of confinements until 1954, and of livebirths from 1955. (The net difference in these is small since biases due to stillbirths and multiple births are opposite in direction.) Registration is by age (in years) of mother rather than by year of birth of mother, so that the data refer to two halves of two adjacent birth cohorts. Conversion to birth cohort data is illustrated in Figure 5.1. It is assumed for example that the $b_1 + b_2$ births that occur to women aged 17 years in the interval t-4 to t-3 years (that is aged 20 or 21 years at time t) comprise half of the births in that interval to women aged 20 years at time t and half of those in the same interval to

Figure 5.1: Lexis diagram illustrating the difference between data by age of mother and by year of birth of mother



women aged 21 years at time t. Furthermore, it is assumed that within the interval the distribution of births over time is uniform for each birth cohort, so that the square area in Figure 5.1 can be assumed equal to the parallelogram bounded by the dotted lines and the interval t-4 to t-3. The tabulated data are thus assumed to refer to single year cohorts of women centred on 1 January: at time t, therefore, these women are on average exactly full years of age. Since the data are tabulated by single years of age, it is possible to choose exact ages 15, 20, ... 50, that is endpoints.

Complete data are available for the birth cohorts of 1870/71 to 1915/16. Taking cohorts at five year intervals from 1870/71 thus gives 10 complete cohort fertility schedules. These data are reproduced in Appendix 5.4 (Table A5.4.2).

The model was fitted to all eight datapoints and to progressively fewer, as for the simulated data. The resulting estimates of completed fertility are shown in Table 5.4, along with their per cent errors. It is seen that the errors for cohorts 1870/71 to 1900/01 are of roughly similar size to those for the simulated data. For later cohorts, larger errors in the estimate of F were obtained and, as seen in the full results in Appendix 5.5 (Table A5.5.2), the fits are not so good. In addition, unlike the estimates for the simulated data, there is no consistent pattern in the sign of the errors.

Table 5.4: Estimates of completed fertility for Swedish data

Points included	Cohort 1870/71	1875/76	1880/81	1885/86	1890/91	1895/96	1900/01	1905/06	1910/11	1915/16
15 to 50	3.70674	3.52359	3.19038	2.89289	2.52178	2.14773	1.88448	1.83757	1.89434	2.00564
15 to 45	3.71329	3.52717	3.19193	2.89639	2.51731	2.14421	1.88348	1.84759	1.90969	2.01088
15 to 40	3.72509	3.53102	3.18907	2.90526	2.53430	2.14266	1.87938	1.88209	1.98023	2.03164
15 to 35	3.68369	3.54486	3.14509	2.88957	2.57408	2.13766	1.86006	1.80791	2.31204	2.15018
15 to 30	3.61762	3.44894	3.34757	2.72492	2.68178	2.11643	1.83018	1.67456	2.10980	3.03656
Actual	3.6994	3.5197	3.1878	2.8884	2.5178	2.1492	1.8846	1.8262	1.8739	1.9970
					Per cent error in estimate					
15 to 50	.20	.11	.08	.16	.16	-.07	-.01	.62	1.09	.43
15 to 45	.38	.21	.13	.28	-.02	-.23	-.06	1.17	1.91	.70
15 to 40	.69	.32	.04	.58	.66	-.30	-.28	3.06	5.67	1.73
15 to 35	-.42	.71	-1.34	.04	2.24	-.54	-1.30	-1.00	23.38	7.67
15 to 30	-2.21	-2.01	5.01	-5.66	6.51	-1.52	-2.89	-8.30	12.59	52.06

Such changeability in the size and sign of the errors is due to the transitional nature of the data. The first five cohorts exhibit small errors of variable sign indicating differential rates of declining fertility for age groups within cohorts. The consistently negative errors obtained for cohorts 1895/96 and 1900/01 indicate that late fertility is higher than expected from the earlier data, even when relatively complete. These cohorts mark the beginning of the transition from declining to increasing fertility which continues over the next three cohorts. The much larger errors for the final cohorts can be attributed to this transition, and their positive sign indicates that late fertility is lower than expected, especially from early experience. Relating these results to the data in Appendix 5.4, it is seen that consistent underestimation of F occurs when early fertility is high in relation to late fertility, and that consistent overestimation occurs when early fertility is low in relation to late fertility. Though this may at first appear contradictory, it is seen from the α and β estimates in Table 5.5 that these results are entirely consistent, and are in line with those for the simulated data. For the cohorts 1895/96 and 1900/01, the β estimates obtained for less than complete data are greater than those obtained for the complete experience of the cohort, indicating smaller variances and correspondingly lower levels of fertility. For the cohorts of 1910/11 and 1915/16, smaller β estimates are obtained for incomplete than for complete

Table 5.5: Estimates of α and β for Swedish data

Points included	1870/71	1875/76	1880/81	1885/86	1890/91	1895/96	1900/01	1905/06	1910/11	1915/16
					α estimates					
15 to 50	-.46238	-.39820	-.31130	-.23227	-.14929	-.05934	-.05210	-.16434	-.23176	-.16033
15 to 45	-.46285	-.39843	-.31122	-.23286	-.14776	-.05758	-.05141	-.16904	-.23698	-.16267
15 to 40	-.46377	-.39884	-.31094	-.23499	-.15332	-.05657	-.04930	-.18549	-.26247	-.16957
15 to 35	-.45685	-.40161	-.30030	-.23033	-.16873	-.05398	-.03633	-.14580	-.38528	-.21971
15 to 30	-.44435	-.38083	-.35090	-.17488	-.20753	-.04407	-.01829	-.06632	-.31418	-.48914
					β estimates					
15 to 50	1.11871	1.15916	1.17507	1.20326	1.25134	1.27102	1.18324	1.09061	1.18188	1.39638
15 to 45	1.11452	1.15626	1.17395	1.19961	1.25615	1.27612	1.18484	1.07745	1.16080	1.38798
15 to 40	1.10730	1.15368	1.17612	1.19196	1.23733	1.27784	1.19069	1.03881	1.07891	1.35388
15 to 35	1.12338	1.14765	1.20045	1.20264	1.20724	1.28298	1.21046	1.10021	0.89242	1.23127
15 to 30	1.14145	1.17583	1.13412	1.27235	1.15836	1.29659	1.23159	1.18743	0.95795	0.92643

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15 to 30	1.14145	1.17583	1.13412	1.27235	1.15836	1.29659	1.23159	1.18743	0.95795	0.92643

data, indicating larger variances and higher levels of fertility. As for the simulated data, increases in the estimate of completed fertility for a particular cohort are associated with decreases in α and/or β, and vice versa.

The negative values of α indicate a later pattern of fertility than in the standard; and values of β greater than 1 indicate a more peaked pattern. Over time the Swedish pattern moves towards earlier fertility until 1900/01 after which α temporarily decreases before rising again in 1915/16, and towards more peaked fertility, again except for a reversal of this trend in 1900/01 and 1905/06. The first six cohorts therefore exhibit a transition in which the decrease in fertility at older ages is very pronounced, so that the variance is reduced and the mean is pushed back towards earlier ages. In absolute terms, early fertility does not change appreciably over this period, but its relative share increases considerably. The reversal of this trend occurs for the cohorts of 1900-1911 where early fertility is decreasing and late fertility increasing, so that both the mean and variance increase, causing α to decrease and β to increase. The cohort of 1915/16 exhibits increasing fertility at younger ages and decreasing fertility at older ages, leading to both increased α and β estimates. Increases in α and β are thus shown to occur when fertility is both declining and rising. This is because the pattern of fertility is moving in a similar direction: for the decline the dominant reduction is at older ages, and for the rise in fertility the dominant increase is at younger ages. The effect on the relative proportions by age is similar.

Native White Women in the United States

These data are drawn from Whelpton (1954, Table A) and relate to white women who were born in the United States of America. They are birth registration data and are tabulated by single year birth cohorts of women, each year being centred on 1 January. Thus, women born between 1 July 1900 and 30 June 1901 are on average exactly 15 years old on 1 January 1915. These are therefore endpoint data.

The data are not complete for any cohort, in that datapoints 15 and 50 are not included: the data being at age 16 and finish at age 47. The maximum amount of information available for any cohort is therefore the six datapoints from age 20 to age 45. This information exists for the birth cohorts of 1899/1900 to 1904/05. The cumulative fertility rates appear in Appendix 5.4 (Table A5.4.3).

The model was fitted to all available datapoints and to progressively fewer. The resulting estimates of completed fertility and the parameters P, Q, α and β, along with the measure of goodness of fit, are shown in Appendix 5.5 (Table A5.5.3). Since actual completed fertility is not known, estimates of cumulative fertility at exact age 45 are used to assess the model. Table 5.6 gives these estimates and their percentage errors. In all cases the errors are negative and with only one exception (1902/03) increase in size as fewer datapoints are involved. Early fertility thus indicates a lower level than is the case, because late fertility is higher than expected. The size of the errors is similar to previous results for fits to data truncated at the same age, despite the loss of information at age 15.

Estimates of α and β appear in Table 5.7; the positive values of α and βs of more than 1 indicate an earlier and more peaked pattern of fertility than in the standard. Within cohorts, α and β change in the opposite direction to F, as for the simulated and Swedish data. The downward trend in β over time is contrary to earlier findings in that though completed fertility is falling, the variance is increasing. It is seen from the data that this is achieved by a greater reduction in fertility at ages 25 to 34 than at younger and older ages. The increase in α over the first four cohorts reflects the greater proportion of fertility achieved by age x_{os} (approximately 25 years), also a result of reduced fertility at 25-34. For the final two cohorts, early fertility falls sufficiently for its proportion to be reduced as well.

Table 5.6: Estimates of cumulative fertility at exact age 45 for US data

Points included	Cohort 1899/1900	1900/01	1901/02	1902/03	1903/04	1904/05
20 to 45	2.60754	2.48206	2.43109	2.42810	2.39205	2.34810
20 to 40	2.59764	2.46785	2.41378	2.42935	2.37420	2.33695
20 to 35	2.55157	2.41962	2.35660	2.36475	2.32659	2.28236
Actual	2.614	2.492	2.443	2.439	2.404	2.355
	Per cent error in estimate					
20 to 45	-0.25	-0.40	-0.49	-0.45	-0.50	-0.29
20 to 40	-0.63	-0.97	-1.20	-0.40	-1.24	-0.77
20 to 35	-2.39	-2.90	-3.54	-3.04	-3.22	-3.08

Table 5.7: Estimates of α and β for US data

Points included	Cohort 1899/1900	1900/01	1901/02	1902/03	1903/04	1904/05
			α estimates			
20 to 45	.06610	.11331	.14305	.14481	.14268	.12627
20 to 40	.07150	.12234	.15472	.14455	.15490	.13387
20 to 35	.09716	.15268	.19298	.18736	.18772	.17098
			β estimates			
20 to 45	1.31660	1.31132	1.28663	1.27405	1.26447	1.22886
20 to 40	1.32877	1.32963	1.30917	1.28470	1.28760	1.24298
20 to 35	1.37352	1.37986	1.37064	1.34401	1.33760	1.29833

Canadian Data

These data are part of a series of Canadian fertility rates for cohorts of women born in 1911 to 1947 (Romaniuk, unpublished). Complete data (to exact age 49) are available for only the first six cohorts, 1911-1916, and these appear in Appendix 5.4 (Table A5.4.4). The data are by single years of age, cumulated to exact ages. In fitting the transformed Gompertz model, end-point data are used.

Since the data are cumulated to exact age 49, only the first seven datapoints were used in the fitting procedure. Again, the model was fitted to fewer and fewer points. The estimates of completed fertility appear in Table 5.8. The per cent errors are based on observed fertility at age 49, though the model assumes that completed fertility is achieved at exact age 50. The error involved in this discrepancy is very small, however, since the additional births achieved during this last year of childbearing would not exceed .001 births per woman.

The positive sign of the errors indicates that earlier fertility points towards higher levels than are observed. This is the same situation as that observed for the same birth cohorts in Sweden, where overestimates of F occur when early fertility is low in relation to later fertility. The large size of the errors, in

Table 5.8: Estimates of completed fertility for Canadian data

Points included	Cohort 1911	1912	1913	1914	1915	1916
15 to 45	2.74907	2.79519	2.89908	2.93725	2.90554	2.89575
15 to 40	2.80025	2.84312	2.94468	2.97726	2.93891	2.92208
15 to 35	2.95389	2.98878	3.07450	3.09464	3.01232	2.95201
15 to 30	3.14122	3.31050	3.44279	3.57424	3.37544	3.25764
Actual*	2.720	2.767	2.873	2.913	2.885	2.879
			Per cent error in estimate			
15 to 45	1.07	1.02	0.91	0.83	0.71	0.58
15 to 40	2.95	2.75	2.49	2.21	1.87	1.50
15 to 35	8.60	8.02	7.01	6.24	4.41	2.54
15 to 30	15.49	19.64	19.83	22.70	17.00	13.15

* Value for exact age 49.

comparison with earlier results, and the correspondingly poorer fits (see Appendix 5.5, Table A5.5.4) result from this pattern of fertility. In addition, the change from increasing to decreasing completed fertility suggests the presence of some instability of the age specific rates.

Estimates of α and β are shown in Table 5.9, and indicate a later and more peaked pattern of fertility than in the standard. Changes within cohorts are consistent with earlier results in that they are opposite in direction to changes in F. The trend in β over time is one of increase, so that the variance is reduced over time. For the first four cohorts, this is the reverse of the US finding (of decreasing F and increasing variance) and, like that finding, is not generally expected. The Canadian result is due to the continued reduction of fertility at ages less than 20 coupled with increases at ages 20 to 29. The trend in α, again the reverse of the US experience, is one of decrease and then increase. This is primarily governed by the level of fertility: as seen from the data, F(25) is rising steadily over the period whilst completed fertility rises sharply, levels somewhat and then declines.

Conclusions

The transformed Gompertz model has been shown to produce good estimates of completed fertility, even comparison with earlier results, and the correspondingly poorer fits (see Appendix 5.5, Table A5.5.4) result from this pattern of fertility. In addition, the change from increasing to decreasing completed fertility suggests the presence of some instability of the age specific rates.

Estimates of α and β are shown in Table 5.9, and indicate a later and more peaked pattern of fertility than in the standard. Changes within cohorts are consistent with earlier results in that they are opposite in direction to changes in F. The trend in β over time is one of increase, so that the variance is reduced over time. For the first four cohorts, this is the reverse of the US finding (of decreasing F and increasing variance) and, like that finding, is not generally expected. The Canadian result is due to the continued reduction of fertility at ages less than 20 coupled with increases at ages 20 to 29. The trend in α, again the reverse of the US experience, is one of decrease and then increase. This is primarily governed by the level of fertility: as seen from the data, F(25) is rising steadily over the period whilst completed fertility rises sharply, levels somewhat and then declines.

Conclusions

The transformed Gompertz model has been shown to produce good estimates of completed fertility, even

Table 5.9: Estimates of α and β for Canadian data

Points included	Cohort					
	1911	1912	1913	1914	1915	1916
			α estimates			
15 to 45	-.15156	-.16629	-.18700	-.17971	-.16794	-.15017
15 to 40	-.16826	-.18093	-.19944	-.19078	-.17748	-.15813
15 to 35	-.21894	-.22752	-.23807	-.22637	-.20110	-.16835
15 to 30	-.27231	-.31428	-.33349	-.34721	-.30067	-.25890
			β estimates			
15 to 45	1.13058	1.15071	1.17507	1.20065	1.21580	1.23052
15 to 40	1.08897	1.11149	1.13718	1.16649	1.18629	1.20693
15 to 35	1.01132	1.03633	1.07027	1.10250	1.14377	1.18765
15 to 30	0.95842	0.94868	0.96841	0.97314	1.03142	1.08241

when the data are truncated at age 30. The data used in these tests are not typical of high fertility populations and are thus not entirely appropriate for use with the model. These results therefore provide a somewhat conservative indication of the performance of the model.

The four sets of data analysed in this chapter cover a variety of fertility patterns and trends. Examination of the three parameters of the model in conjunction has shown that for populations undergoing a fertility transition, in that the trend in the level changes direction, large errors may occur in the estimation of F. Where fertility is declining and there is no indication of change, the model has been shown to perform well.

CHAPTER 6

APPLICATION OF THE MODEL TO MATERNITY HISTORY DATA

Introduction

This chapter describes the analysis of cohort fertility collected in the form of maternity histories. The fitting procedure already described in Chapter 5 is used throughout. The only necessary change is that the 'midpoint' values of the standard, rather than the endpoint values, are used because the data derived from maternity histories are average parities for five year age groups and refer to ages approximately in the middle of the age groups. The data analysed here are from high fertility populations for which the standard was developed.

Maternity History Data

The four sets of data discussed in this chapter were all collected in retrospective surveys in the form of maternity histories. Such data are presented as the average number of births per woman for each five year period before the survey for each 5 year age group of women at the time of the survey. The lexis diagram in Figure 6.1 illustrates the cohorts of women and exact periods of time to which the data refer. For example,

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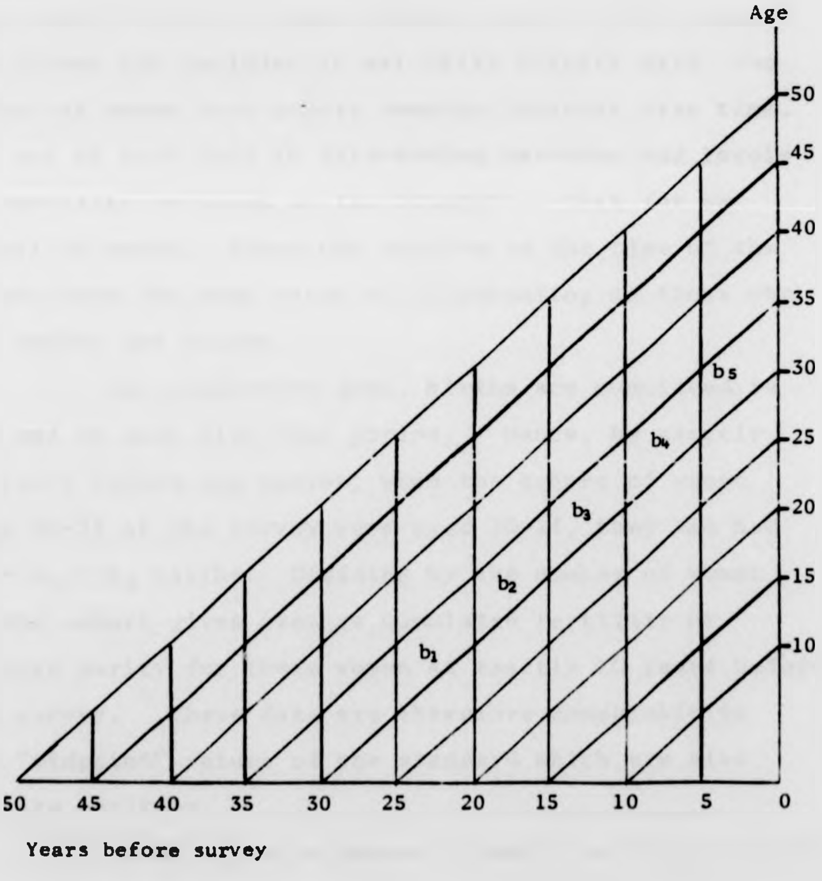
Figure 6.1: Lexis diagram illustrating number of births by time period to cohorts of women



the cohort of women aged 30-34 at the time of the survey entered the childbearing period (at age 10) 20 to 24 years before the survey. By exactly 20 years before the survey these women had had b_1 births; during the next five years they had a further b_2 births and so on. Dividing the b_is by the number of women in the cohort gives the average number of births per woman for that cohort for the relevant 5 year period before the survey. Since only births to women who are alive at the time of the survey are included in maternity history data, the number of women in a cohort remains constant over time. The use of such data in determining patterns and levels of fertility is based on the assumption that for any cohort of women, those who survive to the time of the survey have the same rates of childbearing as those who die before the survey.

For cumulative data, births are cumulated to the end of each five year period. Hence, by exactly 10 years before the survey, when the cohort of women aged 30-34 at the survey were aged 20-24, they had had $b_1 + b_2 + b_3$ births. Dividing by the number of women in the cohort gives average cumulated fertility or average parity for these women at exactly 10 years before the survey. These data are therefore comparable to the "midpoint" values of the standard which are also average parities.

In using the midpoint standard to fit the transformed Gompertz model to average parity data, the assumption is made that the ages to which the average parities refer are the same for the standard and observed data. Clearly, this is not strictly true since different patterns of fertility lead to different ages of average parities, especially at very young and very old ages. The fact that age 10-14 is omitted from the fitting procedure and that age 45-49 is rarely used, reduces the effect of this discrepancy, though the error involved is small and is considered unimportant in relation to reporting errors and biases.

The nature of reporting errors

There are several types of reporting errors that may exist in maternity history data. Errors in the reported ages of women are perhaps the most obvious, and may be related to parity in that women with more children than average for their age may be reported (often by the interviewer) as older than their true age. Such biases will distort the level and trend in fertility. Omissions of births may also occur, especially for older cohorts, again distorting levels and trends. In addition, the pattern of omissions within cohorts may be significant, leading to apparent increases in rates over time at young ages, for example, if children who have grown up and left the household are omitted on a large scale.

Other errors relate to the timing of the reported number of births, that is the distribution of births over the childbearing period, and it is for the detection and correction of these errors that the transformed Gompertz model has been developed. Brass (1971) has identified two types of timing errors: the first is errors in the size of the reference period, and the second is errors in the location of the reference period. Errors in the size of the reference period occur for all women, and may involve the shortening or lengthening of a particular period before the survey. For example, births actually occurring 0 to 6 years ago might be reported as having occurred 0 to 5 years ago, a lengthening of the period. This might be accompanied by a shortening of the previous 5 year reference period to 6 to 9.5 years, the effect operating at both ends of the period. Errors in the location of the reference period occur to different extents for different cohorts of women. There may be a general tendency for women to push their later childbearing period further into the past as they become older, or to bring forward early births, again increasingly with their own age. The combination of these two types of timing errors can be quite complex, and is further complicated by the errors in the ages of the women and by the pattern of omissions discussed above. The effects on fertility may be in similar or opposite directions and it is impossible to identify anything but the major biases in the data.

In his model of event misplacement in maternity histories, Potter (1977) assumes that early births are moved forward in time and that intervals between events are exaggerated, whilst recent events are correctly reported. There is thus a heaping of events in the middle of the childbearing period experienced. Potter demonstrates that this model of event misreporting leads to an apparent or overestimated decline in fertility. He also produces evidence to show that such a pattern of event misplacement occurs in data from Bangladesh and El Salvador.

Method of Fitting

The chosen method of fitting the model to maternity history data is the method of least squares. The technique is chosen for its relative simplicity and for its robustness in a variety of situations. Different sets of data present a number of possible error structures, knowledge of which is not usually available in advance. In the kinds of populations studied here, it is likely that reporting errors and biases will be of greater magnitude than random sampling errors. It is therefore preferable that the fitting technique be robust to such biases, rather than be based on assumptions of randomness. The least squares method with equal weights (that is, unweighted) is extremely robust, and though it may not be the optimum for any given situation, in most cases its results will not differ appreciably from the optimum. In addition, in situations where sampling errors are the main consideration so that maximum likelihood estimation might be more appropriate, the least squares estimates will not be very different from maximum likelihood estimates if the optimum weights are not too variable. (By choosing weights equal to the optimum weights, the two methods are identical.) With the presence of biases in the data, the ability to choose weights afforded by the least squares technique is of advantage.

Choice of weights

The choice of weights involved in the fitting procedure depends on the way in which the data were collected, on the form in which they are used and on the purposes of the fitting exercise. Data for cohorts of women collected in retrospective surveys where memory problems occur warrant different weights than those collected by registration. In the former situation, there is a case for giving greater weight to the more recent time period on the grounds that memory errors are of considerably less importance for this period. In the case of registration data, where there is no reason to favour any time period with respect to accuracy of reporting, equal weighting might apply. Current fertility, such as births in the past year, also collected by retrospective survey or by registration, should be equally reliably reported at all ages so that equal weights would be appropriate.

Cumulated rates obviously require different weights to age specific rates. Registration data are equivalent to age specific rates such that the equal weights apply to these rates rather than to cumulated rates. Where data on children ever born are collected for cohorts of women, these cumulated values warrant the larger weight because of their current status. Recent age specific values might also warrant greater weight than those for earlier periods.

The purpose of the fitting exercise might also influence the choice of weights. The detection of errors and graduation of the data may require extra weight being assigned to more reliable points, as discussed above. For extrapolation, however, it is desirable that the point of departure be as accurate as possible, and this might be achieved by a different set of weights.

For data obtained in maternity histories for high fertility populations, there are advantages in using cumulative rates. The process of cumulation tends to iron out random sampling errors so that they become decreasingly important with age. At the same time, any systematic biases that exist as a result of reporting and timing errors are exaggerated by cumulation so that their presence is easier to detect. Data collected retrospectively over as long as the entire reproductive period as in maternity histories, are generally affected more by bias than by random error, so that it is appropriate to concentrate on the bias in fitting.

For the initial fitting exercise equal weights (with the omission of age 10-14) were used. This choice of weights was largely preliminary and exploratory. The procedure is well-tried and robust, giving a set of reasonable results which, though not necessarily the best in determining the level of fertility, provide some guidance in the assessment of other results. It is these results that are directly comparable with those presented in Chapter 5.

The model was also fitted to the same sets of data using different weights. An infinite weight was given to the latest report for each cohort with equal weights ($w(x) = 1$) at earlier ages, (except that the 10-14 age group was again omitted ($w(10\text{-}14) = 0$) because of its very low fertility being subject to large errors). This is based on the hypothesis that the reporting of total number of births at the time of the survey is accurate, and that only the distribution of these births is in error. This takes no account of omissions, as indeed the model is not intended to so do. It is this latter weighting system that should be used both for graduation and for the prediction of the final level of fertility. The latest reported point is the most accurate and serves the dual purpose of providing both a total to be distributed over past time and point of departure for prediction.

The fitting procedure

The fitting procedure is essentially the same as that used in Chapter 5. The 'midpoint' values of the standard are used and there are two weighting systems as already discussed. The initial parameter estimates are calculated as shown in Appendix 5.2 from estimates of the ordinary Gompertz parameters for both the standard and the observed data. The use of the method of selected points with the observed average parities involves an extra approximation because the points are not strictly equidistant. This together with the reporting errors in the data leads to less accurate initial estimates than those obtained for better quality endpoint data. This is not important, however: for the results reported here convergence has been reached without difficulty and it is likely that the accuracy of the initial estimates is in excess of that actually required.

Bangladesh Fertility Survey Data

The Bangladesh Fertility Survey was conducted in 1975 as part of the World Fertility Survey programme. Data on fertility were collected by means of detailed maternity histories for a sample of 6513 ever-married women. These were converted to fertility rates for all women from knowledge of the proportions of women ever-married. Full details of the survey are given in Bangladesh (1978).

Fertility rates as reported appear in Table 6.1. Current total fertility is about 2.0 less than that for the preceding 5 year period, though this latter period has reported rates above age 20 which are in excess of any other period and are too high. Current rates are somewhat lower than past rates, indicating either a real decline in fertility or under-reporting. Since the adjacent period clearly suffers from over-reporting, the latter explanation seems at this stage more plausible.

The transformed Gompertz model was fitted to the data in the manner described in Chapter 5, using the two weightings discussed above. Estimates of the three parameters of the model for the cohorts aged 30 to 44 at the survey appear in Table 6.2; (full results are given in Appendix 6.1). The cohort aged 45-49 is obviously badly affected by omissions (see Table 6.1) and provides no useful information about the level of fertility.

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Table 6.1: Average births per woman; Bangladesh Fertility Survey, 1975

Cohort: age at survey	Period: years before survey							
	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
10-14	.0080							
15-19	.5450	.0400						
20-24	1.4430	.8445	.0565					
25-29	1.4555	1.8005	.8765	.0480				
30-34	1.2510	1.8005	1.6855	.8825	.0665			
35-39	.9240	1.6585	1.7150	1.4985	.7965	.0430		
40-44	.5370	1.2040	1.5135	1.5245	1.5405	.7540	.0745	
45-49	.1735	.7310	1.1090	1.2120	1.3695	1.2400	.5895	.0420

Cumulated within period

10-14	.0080	.0400	.0565	.0480	.0665	.0430	.0745	.0420
15-19	.5530	.8845	.9330	.9305	.8630	.7970	.6640	
20-24	1.9960	2.6850	2.6185	2.4290	2.4035	2.0370		
25-29	3.4515	4.4855	4.3335	3.9535	3.7730			
30-34	4.7025	6.1440	5.8470	5.1655				
35-39	5.6265	7.3480	6.9560					
40-44	6.1635	8.0790						
45-49	6.3370							

Cumulated to exact years before survey

	0	5	10	15	20	25	30	35
10-14	.0080							
15-19	.5850	.0400						
20-24	2.3440	.9010	.0565					
25-29	4.1805	2.7250	.9245	.0480				
30-34	5.6860	4.4350	2.6345	.9490	.0665			
35-39	6.6355	5.7115	4.0530	2.3380	.8395	.0430		
40-44	7.1480	6.6110	5.4070	3.8935	2.3690	.8285	.0745	
45-49	6.4665	6.2930	5.5620	4.4530	3.2410	1.8715	.6315	.0420

Table 6.2: Estimates of F, α and β; Bangladesh Fertility Survey 1975

Cohort: age at survey	Parameter estimates F	α	β
	a) equal weights		
30-34	7.62113	.25539	.93396
35-39	7.79200	.12603	.90761
40-44	7.53612	.13693	.89806
	b) infinite weight to last point		
30-34	6.99541	.39254	1.11766
35-39	7.86493	.12325	.84406
40-44	7.66322	.15594	.73514

The cohorts aged 25-29 and younger do not provide sufficient data for the estimation of the parameters.

The estimates of the level of fertility are fairly consistent for the cohorts aged 30-34, 35-39 and 40-44, and are in broad agreement with the level of about 7.5 arrived at by a substantial review of all available evidence on fertility in Bangladesh during a workshop on Bangladesh in April 1979 (U.S. National Academy of Sciences, to be published). The α and β estimates exhibit greater variation, but it is unlikely that the variation in the pattern suggested by these parameters is real. The α estimates imply that the proportion of fertility achieved by age 25 (strictly, x_{os} = 24.9 years) rose from 41 to 51 per cent over the space of 5 years (cohorts aged 35-39 to 30-34). Even if fertility up to age 25 were to remain constant, this would require that fertility after age 25 be reduced by a third such that completed fertility be reduced by 20 per cent. In the event of declining fertility, however, it is also likely that early fertility would decline; if this were the case, α implies even greater reductions of later and completed fertility. Such reductions in fertility are not borne out by the F estimates.

The variation in the β estimates is equally unlikely, implying a rapid change in the pattern of fertility from that of a flat distribution with a large variance to one of a more peaked distribution with smaller variance. Again, such a transition would generally be accompanied by a decline in the level of fertility, and the suggestion is not supported by the F estimates.

The fitted values of cumulative fertility obtained under the second set of weights are shown in Table 6.3. The differences between observed and fitted rates show no systematic deviation with regard to age, indicating that the model is appropriate for the data. The age specific rates in Table 6.4 indicate the same pattern of deviation with regard to time for the cohorts aged 35-39 and 40-44 at the survey. The fitted values suggest that the number of births reported to have occurred during the past 5 years is too low, whilst the numbers in the preceding years are too high. For both cohorts, the most serious over-reporting occurs at ages 30-34: this represents a greater timing error for the 40-44 cohort than for the 35-39 cohort. The fitted age specific values for these two cohorts also suggest that births to these women at the beginning of childbearing have been reported as occurring closer to the survey date. There is thus evidence of a shortening of the reference period operating at both ends.

For the cohort aged 30-34 at the survey, the opposite pattern of deviations is obtained, suggesting over-reporting in the recent past (slight) and at very

Table 6.3: Observed and fitted cumulative fertility rates; Bangladesh Fertility Survey, 1975

	Cohort: age at survey					
	30-34			35-39		
Age	observed (1)	fitted (2)	difference (1-2)	observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.06650)	(.00739)	(.05911)	(.04300)	(.04856)	(-.00556)
15-19	.94900	.73341	.21559	.83950	.87349	-.03399
20-24	2.63450	2.68658	-.05208	2.33800	2.48967	-.15167
25-29	4.43500	4.43926	-.00426	4.05300	4.08207	.02907
30-34	5.68600	5.68600	-	5.71150	5.47477	.23673
35-39		6.48286		6.63550	6.63550	-
40-44		6.89202			7.48446	
45-49		6.98998			7.82314	

	Cohort: age at survey		
	40-44		
	observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.07450)	(.15077)	(-.07627)
15-19	.82850	1.15633	-.32783
20-24	2.36900	2.61258	-.24358
25-29	3.89350	3.96210	-.06860
30-34	5.40700	5.17086	.23614
35-39	6.61100	6.25206	.35894
40-44	7.14800	7.14800	-
45-49		7.58716	

*Age 10-14 not included in fitting procedure.

Table 6.4: Observed and fitted age specific fertility rates; Bangladesh Fertility Survey, 1975

	Cohort: age at survey					
	30-34			35-39		
Age	observed (1)	fitted (2)	difference (1-2)	observed (1)	fitted (2)	difference (1-2)
15-19	.88250	.72602	.15648	.79650	.82493	-.02843
20-24	1.68550	1.95317	-.26767	1.49850	1.61618	-.11768
25-29	1.80050	1.75268	.04782	1.71500	1.59240	.12260
30-34	1.25100	1.24674	.00426	1.65850	1.39270	.26580
35-39				.92400	1.16073	-.23673

	Cohort: age at survey		
	40-44		
Age	observed (1)	fitted (2)	difference (1-2)
15-19	.75400	1.00556	-.25156
20-24	1.54050	1.45025	.09025
25-29	1.52450	1.34952	.17498
30-34	1.51350	1.20876	.30474
35-39	1.20400	1.08120	.12280
40-44	.53700	.89594	-.35894

young ages with quite serious under-reporting at age 20-24. This result seems implausible: the fitted age specific fertility for age 20-24 of 1.95 is even higher than the reported rate of 1.80 in the period 5-10 years ago, already stated to be too high. Such a high rate at this age also contradicts the level estimate of only 7.00. Examination of the α and β estimates shows large differences from those obtained for the older cohorts, and from those obtained for the first set of weights (equal weighting), suggesting that the fitted pattern of fertility is not good. The implausibility of such a large variation in α and β over cohorts has already been discussed and it is likely that the cohort provides insufficient information for reliable estimation. Given the relationship between the parameters of the model, it is probable that these unlikely α and β estimates have resulted in an underestimate of F. This seems to be case, though the amount is not as serious as the pattern parameters might suggest.

The graduated fertility rates appear in Table 6.5. Rates for cohorts aged 35-39 and 40-44 are those obtained directly by fitting the model. The remainder have been constructed from the fit obtained for the cohort aged 35-39, since this is less affected by omissions than the cohort aged 40-44. In addition, for cohorts aged 30+ there is no evidence of any significant trend in marriage patterns which would affect the pattern of fertility:

Table 6.5: Graduated fertility rates by period; Bangladesh Fertility Survey, 1975

Cohort: age at survey	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
	Period: years before survey							
10-14	.04000							
15-19	.54500	.04000						
20-24	1.52161	.77667	.04572					
25-29	1.63080	1.65514	.84482	.04972				
30-34	1.44645	1.65383	1.67853	.85675	.05044			
35-39	1.16073	1.39270	1.59240	1.61618	.82493	.04856		
40-44	.89594	1.08120	1.20876	1.34952	1.45025	1.00556	.15077	
45-49	.33868	.84896	1.16073	1.39270	1.59240	1.61618	.82493	.04856
Age	Cumulated within period							
10-14	.04000	.04000	.04572	.04972	.05044	.04856	.15077	.04856
15-19	.58500	.81667	.89054	.90647	.87537	1.05412	.97570	
20-24	2.10661	2.47181	2.56907	2.52265	2.32562	2.67030		
25-29	3.73741	4.12564	4.16147	3.87217	3.91802			
30-34	5.18386	5.51834	5.37023	5.26487				
35-39	6.34459	6.59954	6.53096					
40-44	7.24053	7.44850						
45-49	7.57921							
TF	7.64	8.18						

the mean age at first marriage, for those marrying before age 20, is 11.7, 11.7, 11.6 and 11.1 years for cohorts aged 30-34 to 45-49 respectively, and is too low and the change too slight to have any appreciable effect on fertility patterns. For the cohort aged 45-49, the graduated rates for the cohort aged 35-39 have been used without modification, because the reported level is clearly too low at 45-49. The pattern parameters obtained for the cohort aged 30-34 were rejected as unlikely (as discussed above). For this and the cohorts aged 25-29 and 20-24, the graduated rates were constructed using the pattern estimates for the 35-39 cohort (the best available) and the latest report of children ever born for the appropriate cohort. For earlier cohorts, aged less than 20, reported fertility for the cohort aged 15-19 is used.

The rates are cumulated within period in the lower half of Table 6.5. Total fertility rates (TF) for the two most recent five year periods were obtained by extrapolation by means of fitting the model to these graduated rates. (The results of these fits are given in Appendix 6.2.) Taken at face value, these total fertilities imply a current level of 7.64 with a fall of about 0.5 during the last five years. The reported decline of almost 2 births per woman during the last five years has thus been largely attributed to reporting error. The graduated rates at ages 30-34 and 35-39 for the periods 10-14 and 15-19 years before the survey suggest that not all of the reporting error may have been removed, however. In addition, the use of the pattern parameters for cohort 35-39 in obtaining graduated rates for cohorts aged 20 to 29 is not entirely satisfactory because of the possible later pattern of fertility due to the increases in age at marriage for these cohorts. The mean age at first marriage for those marrying before age 20 is 12.5 and 12.3 years for cohorts aged 20-24 and 25-29 respectively, rather higher than the 11.7 years for the cohort aged 35-39. If there has been some movement towards later fertility for these younger cohorts, the reported mean parities should be redistributed away from the very young ages to ages 20 to 29. This would have the effect of increasing graduated fertility in the most recent period in relation to the preceding period, thereby reducing even further the reported decline in fertility over the last five years.

Support for an adjustment of this kind comes from the fit of the model to the graduated rates for the two most recent periods (see Appendix 6.2). The fitted pattern is rather flat for both periods with the largest deviations at the ages (15-19 for the most recent period, and 15-19 and 20-24 for the preceding period) which would be most affected. In the most recent period, an increase in the 20-24 rate (with a possible but smaller increase at 25-29) would result in a higher estimate of β with a correspondingly lower estimate of the 15-19 value. The slight reduction in TF resulting from increased β would be offset by the increase in the graduated cumulative rates. In the preceding period, a decrease in the 15-19 rate accompanied by a possible slight increase at 20-24 would also increase β, probably to such an extent that the massive deviation at age 15-19 would be reduced, and in any event reducing the high estimated late fertility thereby producing, in combination with the reduced graduated F values, a significant reduction in TF. The overall effect would thus be to reduce the decline in fertility over the last five years to about 0.2, but to leave the current estimated total fertility at about 7.6.

Sri Lanka Fertility Survey Data

The Sri Lanka Fertility Survey was also conducted in 1975 as part of the World Fertility Survey programme. Maternity histories were collected for a sample of 6813 ever-married women aged 12-49. Proportions ever-married were used to calculate fertility rates for all women. Full details of the survey are given in Sri Lanka (1978).

For the present analysis, the sample was divided into two on the basis of education. This was to try to gain a clearer picture of possible falls in fertility, and is possible in Sri Lanka because the high level of education provides sufficient numbers in the more educated group. The sample was divided according to level of education attained: those receiving no formal education or up to five years education, 61.6 per cent; and those receiving six or more years education, 38.4 per cent. This division of the sample causes problems in the study of the fertility of the two groups, especially the more educated group. Since educational level has been increasing in Sri Lanka, the older cohorts include a smaller proportion of more educated women than the younger cohorts. This distorts the fertility trends of the two groups because though some women may have attained a higher educational level than they would have had they been born earlier into the same social stratum, their fertility has not changed sufficiently to put them on a par with older educated women. In other words, changes in fertility have not occurred as quickly as changes in education. In addition, the small proportion of educated women in the older, and therefore, smaller, cohorts creates small sample sizes of older educated women. This is shown in Table 6.6.

Sri Lanka: 0-5 years education

Fertility rates as reported by women with no education or up to 5 years education are given in Table 6.7. There is no evidence of omissions by the older

Table 6.6: Distribution of cohorts of ever-married women by educational level; Sri Lanka Fertility Survey, 1975

Cohort: age at survey	Years of education 0-5 n	%	6+ n	%	All n
< 20	117	59.1	81	40.9	198
20-24	508	55.3	410	44.7	918
25-29	695	52.8	621	47.2	1316
30-34	701	57.6	516	42.4	1217
35-39	724	60.9	465	39.1	1189
40-45	702	71.6	278	28.4	980
45+	753	75.7	242	24.3	995
All	4200	61.6	2613	38.4	6813

Table 6.7: Average births per woman; Sri Lanka Fertility Survey, 1975, 0-5 years education

Cohort: age at survey	Period: years before survey						
	0-4	5-9	10-14	15-19	20-24	25-29	30-34
15-19	.0465	.0035					
20-24	.5190	.1570	.0075				
25-29	.9865	.7985	.2770	.0220			
30-34	.9970	1.3275	1.1210	.3835	.0215		
35-39	.7940	1.2785	1.4675	1.1165	.4690	.0390	
40-44	.3820	.9200	1.3390	1.4435	1.2060	.3995	.0305
45-49	.1710	.5190	1.0770	1.3920	1.4005	1.1410	.3295
	Cumulated within period						
15-19	.0465	.1605	.2845	.4055	.4905	.4385	.3600
20-24	.5655	.9590	1.4055	1.5220	1.6965	1.5795	
25-29	1.5520	2.2865	2.8730	2.9655	3.0970		
30-34	2.5490	3.5650	4.2120	4.3575			
35-39	3.3430	4.4850	5.2890				
40-44	3.7250	5.0040					
45-49	3.8960						
	Cumulated to exact years before survey						
	0	5	10	15	20	25	30
15-19	.0500	.0035					
20-24	.6835	.1645	.0075				
25-29	2.0840	1.0975	.2990	.0220			
30-34	3.8505	2.8535	1.5260	.4050	.0215		
35-39	5.1645	4.3705	3.0920	1.6245	.5080	.0390	
40-44	5.7205	5.3385	4.4185	3.0795	1.6360	.4300	.0305
45-49	6.0300	5.8590	5.3400	4.2630	2.8710	1.4705	.3295

cohorts. The data suggest that there has been a very rapid decrease in fertility especially at younger ages and over the last 5 years.

Estimates of the parameters of the transformed Gompertz model, obtained from fitting with the two sets of weights, are shown in Table 6.8 (and full results are shown in Appendix 6.1). Since there is no evidence of omissions for the 45-49 cohort, results for this cohort are also presented. There is no clear pattern in the level estimates, nor in α and β, and their erratic nature suggests the presence of reporting errors in the data. The variation between the estimates, and between those obtained for the same cohorts but for different sets of weights, is much smaller than for Bangladesh, however, indicating that the extent of the errors in these data is also less. The results for the cohort 30-34 are somewhat out of line with those for older cohorts and it is likely that their accuracy is impaired by the lack of data points.

The fitted values of cumulative fertility obtained with the second set of weights are shown in Table 6.9. The corresponding age specific rates are shown in Table 6.10. The deviations between the fitted and observed rates are far smaller than those obtained for Bangladesh, indicating that timing errors for Sri Lanka are less of a problem. There is , however, a similar pattern in the deviations. The fitted values for ages 35+ suggest under-reporting of births in the last 5 years, and in the last 10 years for the cohort

Table 6.8: Estimates of F, α and β; Sri Lanka Fertility Survey, 1975, 0-5 years education

Cohort: age at survey	Parameter estimates F	α	β
	a) equal weights		
30-34	5.23391	.12439	1.00861
35-39	5.98055	.07468	.97344
40-44	5.93519	.08328	1.04474
45-49	6.07164	-.03890	1.01536
	b) infinite weight to last point		
30-34	5.01931	.14725	1.11684
35-39	6.10450	.07104	.87919
40-44	5.87777	.09940	1.02784
45-49	6.05348	-.01612	.91922

Table 6.9: Observed and fitted cumulative fertility rates; Sri Lanka Fertility Survey, 1975, 0-5 years education

	Cohort: age at survey					
		30-34			35-39	
Age	observed (1)	fitted (2)	difference (1-2)	observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.02150)	(.00080)	(.02070)	(.03900)	(.01914)	(.01986)
15-19	.40500	.28184	.12316	.50800	.55108	-.04308
20-24	1.52600	1.47785	.04815	1.62450	1.79276	-.16826
25-29	2.85350	2.80649	.04701	3.09200	3.08514	.00686
30-34	3.85050	3.85050	-	4.37050	4.22600	.14450
35-39		4.55348		5.16450	5.16450	-
40-44		4.92444			5.82807	
45-49		5.01431			6.07686	

	Cohort: age at survey					
		40-44			45-49	
Age	observed (1)	fitted (2)	difference (1-2)	observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.03050)	(.00286)	(.02764)	(.00000)	(.00652)	(-.00652)
15-19	.43000	.37790	.05210	.32950	.39098	-.06148
20-24	1.63600	1.68819	-.05219	1.47050	1.56358	-.09308
25-29	3.07950	3.13246	-.05296	2.87100	2.90527	-.03427
30-34	4.41850	4.33044	.08806	4.26300	4.12058	.14242
35-39	5.33850	5.20491	.13359	5.34000	5.11365	.22635
40-44	5.72050	5.72050	-	5.85900	5.79230	.06670
45-49		5.86724		6.03000	6.03000	-

*Age 10-14 not included in fitting procedure.

Table 6.10: Observed and fitted age specific fertility rates; Sri Lanka Fertility Survey, 1975, 0-5 years education

	Cohort: age at survey					
	30-34			35-39		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
15-19	.38350	.28104	.10246	.46900	.53194	-.06294
20-24	1.12100	1.19601	.07501	1.11650	1.24168	-.12518
25-29	1.32750	1.32864	-.00114	1.46750	1.29238	.17512
30-34	.99700	1.04401	-.04701	1.27850	1.14086	.13764
35-39				.79400	.93850	-.14450

	Cohort: age at survey					
	40-44			45-49		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
15-19	.39950	.37504	.02446	.32950	.38446	-.05496
20-24	1.20600	1.31029	-.10429	1.14100	1.17260	-.03160
25-29	1.44350	1.44427	-.00077	1.40050	1.34169	.05881
30-34	1.33900	1.19798	.14102	1.39200	1.21531	.17669
35-39	.92000	.87447	.04553	1.07700	.99307	.08393
40-44	.38200	.51559	-.13359	.51900	.67865	-.15965
45-49				.17100	.23770	-.06670

aged 45-49, coupled with under-reporting at early ages, so that there is heaping of reported births at around age 30-34. For the cohort aged 30-34, the fitted values suggest under-reporting in the period prior to the survey only, so that births are pushed back to the early years of childbearing.

The graduated fertility rates appear in Table 6.11. Rates for cohorts aged over 30 are those obtained directly from fitting the model. Values for cohorts 20-24 and 25-29 were calculated using the pattern estimates for the 30-34 cohort and reported children ever born at the survey. For earlier cohorts, reported fertility for the cohort aged 15-19 is used. Extrapolation to age 50 to obtain total fertility rates for the two most recent periods was done by fitting the model to these period rates. (Results of these fits are given in Appendix 6.2.) The graduated data suggest a current level of fertility of 4.30 and a decline in the last 5 years of about 1.15. Previous periods, back to 25 years before the survey, also suggest that fertility has been steadily declining, though at a slower rate. The reported trend in fertility has thus been reduced but not removed by the model. This has been achieved by an increase in current rates rather than by a reduction of rates in the preceding period.

Examination of nuptiality trends shows that some of the decline in fertility could be due to increased age at marriage: the mean age at first marriage for those marrying before age 25 is 17.9, 17.3, 16.9, 17.2 and 17.5 years for cohorts aged 25-29 to 45-49 respectively.

Table 6.11: Graduated fertility rates by period; Sri Lanka Fertility Survey, 1975, 0-5 years education

Cohort: age at survey	Period: years before survey 0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
10-14	.00350							
15-19	.04650	.00350						
20-24	.55315	.12998	.00037					
25-29	.95272	.91554	.21513	.00061				
30-34	1.04401	1.32864	1.19601	.28104	.00080			
35-39	.93850	1.14086	1.29238	1.24168	.53194	.01914		
40-44	.51559	.87447	1.19798	1.44427	1.31029	.37504	.00286	
45-49	.23770	.67865	.99307	1.21531	1.34169	1.17260	.38446	.00652
			Cumulated within period					
10-14	.00350	.00350	.00037	.00061	.00080	.01914	.00286	.00652
15-19	.05000	.13348	.21550	.28165	.53274	.39418	.38732	
20-24	.60315	1.04902	1.41151	1.52333	1.84303	1.56678		
25-29	1.55587	2.37766	2.70389	2.96760	3.18472			
30-34	2.59988	3.51852	3.90187	4.18291				
35-39	3.53838	4.39299	4.89494					
40-44	4.05397	5.07164						
45-49	4.29167							
TF	4.30	5.45						

This trend also indicates that use of the pattern parameters for cohort 30-34 to graduate the fertility of younger cohorts is slightly in error. A later, more peaked pattern is indicated for these cohorts, the effect of which would be to increase graduated rates in the most recent period slightly, and possibly reduce those in the preceding period, so that the decline in fertility is further reduced.

The effect of such an adjustment to the cohorts aged 20-24 and 25-29 on the fits to graduated rates in the two most recent periods would be to reduce the large change in the pattern of fertility suggested by the α and β parameters for the two periods. The variability in these parameters is clearly too great (see Appendix 6.2) and the peak in the recent period occurs at the unlikely age 30-34. An increase in fertility at ages 20-24 and 25-29 in the recent period would reduce β and increase α, and increase the total fertility estimate (by more than the increase in early rates) because of the higher estimated late fertility. (The apparent contradiction here between the effect of the same adjustment on the β values for these data and those for Bangladesh arises from the differing locations of the two fertility distributions.) The effect of the adjustment on rates in the period 5-9 years before the survey would be to reduce fertility at 15-19 whilst leaving the 20-24 rate relatively unchanged. This would increase β and reduce α, and would further reduce the total fertility estimate. Thus, the current level of fertility would be increased to at least 4.5 and the decline over the last 5 years reduced to about 0.75.

Sri Lanka: 6+ years education

Births reported by women with six or more years education are shown in Table 6.12. These reported data show a marked decline in fertility over the entire 35 year period, with falls in total fertility of about 0.5 per five year period over the past 10 years to a current level of about 3.5. The very large difference in cohort fertility between cohorts aged 40-44 and 45-49 is probably due to the small sample sizes involved at these ages as already discussed.

Table 6.13 shows the parameter estimates obtained using both sets of weights. Full results appear in Appendix 6.1. Leaving aside values for the 45-49 cohort, the level estimates obtained with the second set of weights show slight increases in fertility for younger cohorts, though this trend is not present in the estimates obtained using equal weights, and is contrary to the reported decline. However, the uniformity of the F estimates for the two sets of results and the modest changes between cohorts suggest that the data are relatively free from reporting errors. The pattern parameter reduce α, and would further reduce the total fertility estimate. Thus, the current level of fertility would be increased to at least 4.5 and the decline over the last 5 years reduced to about 0.75.

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Table 6.12: Average births per woman; Sri Lanka Fertility Survey, 1975, 6+ years education

Cohort: age at survey	Period: years before survey 0-4	5-9	10-14	15-19	20-24	25-29	30-34
15-19	.0405						
20-24	.4590	.0655					
25-29	.8750	.4250	.0780				
30-34	.9800	.9460	.5425	.0955	.0065		
35-39	.7285	1.1385	1.1630	.5670	.0630		
40-44	.3770	.8405	1.1310	1.0135	.5915	.0895	
45-49	.0500	.5080	1.0545	1.3315	1.2855	.6635	.1425
			Cumulated within period				
15-19	.0405	.0655	.0780	.0955	.0695	.0895	.1425
20-24	.4995	.4905	.6205	.6625	.6610	.7530	
25-29	1.3745	1.4365	1.7835	1.6760	1.9465		
30-34	2.3545	2.5750	2.9145	3.0075			
35-39	3.1130	3.4155	3.9690				
40-44	3.4900	3.9235					
45-49	3.5400						
			Cumulated to exact years before survey				
	0	5	10	15	20	25	30
15-19	.0405						
20-24	.5245	.0655					
25-29	1.3780	.5030	.0780				
30-34	2.5705	1.5905	.6445	.1020	.0065		
35-39	3.6600	2.9315	1.7930	.6300	.0630		
40-44	4.0430	3.6660	2.8255	1.6945	.6810	.0895	
45-49	5.0355	4.9855	4.4775	3.4230	2.0915	.8060	.1425

Table 6.13: Estimates of F, α and β; Sri Lanka Fertility Survey, 1975, 6+ years education

Cohort: age at survey	Parameter estimation F	α	β
	a) equal weights		
30-34	4.27207	-.33033	.95477
35-39	4.10442	-.24159	1.24195
40-44	4.18861	-.26770	1.12616
45-49	5.09426	-.26842	1.13834
	b) infinite weight to last point		
30-34	4.28322	-.32004	.93875
35-39	4.26523	-.19372	1.06012
40-44	4.19381	-.23757	1.03856
45-49	5.04123	-.26357	1.16295

estimates show no clear trend. The negative values of α result from the late fertility of the more educated women. As a result of being based on few datapoints, the β value for the cohort aged 30-34 is rather too low, especially in combination with only 25 per cent of fertility achieved by age 25, and has probably resulted in an overestimate of F. The problems associated with selection according to education, discussed above, will also have affected these results, and it is impossible to assess the extent to which this selection factor has counterbalanced a decline in fertility.

Fitted cumulative fertility obtained with the second set of weights appears in Table 6.14. The deviations between observed and fitted age specific fertility in Table 6.15 are small in comparison to those for less educated women, indicating that the data are relatively accurate, as is to be expected. Even so, the pattern of deviations is the same as that for less educated women, except for the cohort aged 30-34, suggesting that the same type of reporting errors occur.

Graduated rates appear in Table 6.16. For cohorts aged over 30, rates obtained from fitting the model are used. For the cohort aged 25-29, rates were calculated using the pattern parameters obtained for the 30-34 cohort and reported children ever born at the time of the survey. Reported rates for the 20-24 cohort are used for cohorts aged less than 25. Total fertilities

Table 6.14: Observed and fitted cumulative fertility rates; Sri Lanka Fertility Survey, 1975, 6+ years education

	Cohort: age at survey					
	30-34			35-39		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.00650)	(.00028)	(.00622)	(.00000)	(.00008)	(-.00008)
15-19	.10200	.09661	.00539	.06300	.09452	-.03152
20-24	.64450	.67643	-.03193	.63000	.78752	-.15752
25-29	1.59050	1.59475	-.00425	1.79300	1.85216	-.05916
30-34	2.57050	2.57050	-	2.93150	2.87101	.06049
35-39		3.43706		3.66000	3.66000	-
40-44		4.05029			4.12856	
45-49		4.26322			4.25680	

	Cohort: age at survey					
	40-44			45-49		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.00000)	(.00008)	(-.00008)	(.00000)	(.00000)	(.00000)
15-19	.08950	.08586	.00364	.14250	.05252	.08998
20-24	.68100	.72641	-.04541	.80600	.77649	.02951
25-29	1.69450	1.74265	-.04815	2.09150	2.12786	-.03636
30-34	2.82550	2.74699	.07851	3.42300	3.44490	-.02190
35-39	3.66600	3.54965	.11635	4.47750	4.40797	.06953
40-44	4.04300	4.04300	-	4.98550	4.91880	.06670
45-49		4.18395		5.03550	5.03550	-

*Age 10-14 not included in fitting procedure.

Table 6.15: Observed and fitted age specific fertility rates; Sri Lanka Fertility Survey, 1975, 6+ years education

	Cohort: age at survey					
	30-34			35-39		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
15-19	.09550	.09633	-.00083	.06300	.09444	-.03144
20-24	.54250	.57982	-.03732	.56700	.69300	-.12600
25-29	.94600	.91832	.02768	1.16300	1.06464	.09836
30-34	.98000	.97575	.00425	1.13850	1.01885	.11965
35-39				.72850	.78899	-.06049

	Cohort: age at survey					
	40-44			45-49		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
15-19	.08950	.08578	.00372	.14250	.05252	.08998
20-24	.59150	.64055	-.04905	.66350	.72397	-.06047
25-29	1.01350	1.01624	-.00274	1.28550	1.35137	-.06587
30-34	1.13100	1.00434	.12666	1.33150	1.31704	.01446
35-39	.84050	.80266	.03784	1.05450	.96307	.09143
40-44	.37700	.49335	-.11635	.50800	.51083	-.00283
45-49				.05000	.11670	-.06670

Table 6.15: Observed and fitted age specific fertility rates; Sri Lanka Fertility Survey, 1975, 6+ years education

	Cohort: age at survey					
	30-34			35-39		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
15-19	.09550	.09633	-.00083	.06300	.09444	-.03144
20-24	.54250	.57982	-.03732	.56700	.69300	-.12600
25-29	.94600	.91832	.02768	1.16300	1.06464	.09836
30-34	.98000	.97575	.00425	1.13850	1.01885	.11965
35-39				.72850	.78899	-.06049

	Cohort: age at survey					
	40-44			45-49		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
15-19	.08950	.08578	.00372	.14250	.05252	.08998
20-24	.59150	.64055	-.04905	.66350	.72397	-.06047
25-29	1.01350	1.01624	-.00274	1.28550	1.35137	-.06587
30-34	1.13100	1.00434	.12666	1.33150	1.31704	.01446
35-39	.84050	.80266	.03784	1.05450	.96307	.09143
40-44	.37700	.49335	-.11635	.50800	.51083	-.00283
45-49				.05000	.11670	-.06670

Table 6.16: Graduated fertility rates by period; Sri Lanka Fertility Survey, 1975, 6+ years education

Cohort: age at survey	Period: years before survey 0-4	5-9	10-14	15-19	20-24	25-29	30-34
15-19	.06550						
20-24	.45900	.06550					
25-29	.79351	.50102	.08323	.00024			
30-34	.97575	.91832	.57982	.09633	.00028		
35-39	.78899	1.01885	1.06464	.69300	.09444	.00008	
40-44	.49335	.80266	1.00434	1.01624	.64055	.08578	.00008
45-49	.11670	.51083	.96307	1.31704	1.35137	.72397	.05252
				Cumulated within period			
15-19	.06550	.06550	.08323	.09657	.09472	.08586	.05260
20-24	.52450	.56652	.66305	.78957	.73527	.80983	
25-29	1.31801	1.48484	1.72769	1.80581	2.08664		
30-34	2.29376	2.50369	2.73203	3.12285			
35-39	3.08275	3.30635	3.69510				
40-44	3.57610	3.81718					
45-49	3.69280						
TF	3.72	4.07					

were obtained by extrapolation using the fit of the model to the graduated period rates, the results of which are given in Appendix 6.2. The reported decline in fertility has been reduced slightly: current total fertility is estimated as 3.72 with a fall of only 0.35 during the past five years. Current fertility has been increased slightly by the model, whilst that during the two preceding periods has been reduced. The general trend of declining fertility reported for the past 35 years has not been changed, however.

Examination of nuptiality trends again suggests that rising age at marriage is partly the reason for declining fertility. The mean age at first marriage for those marrying before 25 years is 20.3, 19.7, 20.0, 19.4 and 19.3 for cohorts aged 25-29 to 45-49 respectively. Again these values indicate that a slightly later and more peaked pattern of fertility than that used is appropriate for the cohort aged 25-29. This would result in an increase in current fertility and slight decrease in the previous period, such that the decline over the past five years would be reduced slightly further.

Again, the fits of the model to the graduated rates (see Appendix 6.2) are examined to determine the total effect of such an adjustment to the graduated fertility for cohort 25-29. For the most recent period, an increase at age 25-29 would decrease β slightly and hence increase total fertility by slightly more than the increase at 25-29. For the preceding period, β would be increased due to a fall in the 20-24 rate, thus further decreasing total fertility slightly. It must be stressed that these effects would be slight, partly because only one cohort would be adjusted and partly because of the better quality of the data. The α and β estimates obtained for the graduated rates are already reasonably uniform, so that there is little room for improvement. Current fertility is thus estimated at about 3.8 with a decline over the past five years of about 0.2.

West New Guinea Data

In addition to the maternity history data available from the World Fertility Survey programme, there are other such data available for West New Guinea. These data were collected in 1961 and 1962 in surveys reported by Groenewegen and van de Kaa (1964-1967). The amalgamated data used here are for a sample of about 19,000 women. The large size of the sample is advantageous because sampling errors are reduced.

Fertility rates reported in the surveys are shown in Table 6.17. The data are not truncated at age 50 years and are cumulated within period to age 50-54. The period rates show fertility to have been rising quite rapidly over the last 15 years with an increase of almost

Table 6.17: Average births per woman; West New Guinea, 1961-62

Cohort: age at survey		Period:	years before survey			
	0-4	5-9	10-14	15-19	20-24	All
15-19	.168					.168
20-24	1.356	.198	.001			1.555
25-29	1.864	1.308	.226			3.398
30-34	1.691	1.667	1.360	.255		4.973
35-39	1.310	1.442	1.576	1.315	.324	5.967
40-44	.647	1.055	1.365	1.407	1.423	6.239
45-49	.102	.453	.938	1.173	1.517	5.996
50-54	.005	.092	.432	.741	1.196	5.728
55-59		.006	.061	.318	.795	5.619
60+			.004	.084	.411	5.625
		Cumulated within period				
15-19	.168	.198	.227	.255	.324	
20-24	1.524	1.506	1.587	1.570	1.747	
25-29	3.388	3.173	3.163	2.977	3.264	
30-34	5.079	4.615	4.528	4.150	4.460	
35-39	6.389	5.670	5.466	4.891	5.255	
40-44	7.036	6.123	5.898	5.209	5.666	
45-49	7.138	6.215	5.959	5.293		
50-54	7.143	6.221	5.963			
	Cumulated to exact years before survey					
	0	5	10	15	20	25
15-19	.168					
20-24	1.555	.199	.001			
25-29	3.398	1.534	.226			
30-34	4.973	3.282	1.615	.255		
35-39	5.967	4.657	3.215	1.639	.324	
40-44	6.239	5.592	4.537	3.172	1.765	.342
45-49	5.996	5.894	5.441	4.503	3.330	1.813
50-54	5.728	5.723	5.631	5.199	4.458	3.262
55-59	5.619	5.619	5.613	5.552	5.234	4.439
60+	5.625	5.625	5.625	5.621	5.537	5.126

2 births per woman, half of which is attributed to the last 5 years.

Estimates of the parameters of the model are given in Table 6.18 for the cohorts aged 30 to 44. (Full results appear in Appendix 6.1.) Older cohorts are clearly affected by omissions, and also suffer from truncation at early ages because the period under study is only 25 years. The level parameters relate strictly to exact age 50, because the model assumes zero fertility after that age. The error involved is obviously small, however, and will not affect the conclusions. There is a trend towards higher fertility for the younger cohorts in both sets of results. The pattern parameters are less consistent between sets of results, though α shows a trend towards later fertility in both. The consistency in β for equal weights is not obtained for the second weighting. The low β value for the cohort aged 30-34, in conjunction with the small proportion (about 26 per cent) of completed fertility assumed by the model to be achieved by age 25, is rather unlikely, and has probably affected the level estimate which is too high.

Fitted cumulative fertility rates, obtained by using the second set of weights, are shown in Table 6.19. The corresponding age specific rates appear in Table 6.20. Cohorts 30-34 and 40-44 exhibit the same pattern of deviations found previously, suggesting that heaping occurs

Table 6.18: Estimates of F, α and β; West New Guinea, 1961-62

Cohort: age at survey	Parameter estimates F	α	β
	a) equal weights		
30-34	7.53636	-.14610	.96523
35-39	7.17161	-.11079	.91631
40-44	6.47343	.01316	.96790
	b) infinite weight to last point		
30-34	8.78789	-.30626	.82267
35-39	7.05181	-.08660	.96043
40-44	6.75141	.02745	.73593

Table 6.19: Observed and fitted cumulative fertility rates; West New Guinea, 1961-62

	Cohort: age at survey					
	30-34			35-39		
Age	Observed (1)	fitted (2)	difference (1-2)	Observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.00000)	(.00497)	(-.00497)	(.00000)	(.00240)	(-.00240)
15-19	.25500	.32422	-.06922	.32400	.32623	-.00223
20-24	1.61500	1.51851	.09649	1.63900	1.61930	.01970
25-29	3.28200	3.18375	.09825	3.21500	3.24453	-.02953
30-34	4.97300	4.97300	-	4.65700	4.75000	-.09300
35-39				5.96700	5.96700	-
40-44					6.76775	
45-49					7.02894	

	Cohort: age at survey		
	40-44		
Age	Observed (1)	fitted (2)	difference (1-2)
(10-14)*	(.00000)	(.07696)	(-.07696)
15-19	.34200	.78464	-.44264
20-24	1.76500	1.98540	-.22040
25-29	3.17200	3.18948	-.01748
30-34	4.53700	4.31799	.21901
35-39	5.59200	5.35848	.23352
40-44	6.23900	6.23900	-
45-49		6.67562	

* Age 10-14 not included in fitting procedure

Table 6.20: Observed and fitted age specific fertility rates; West New Guinea, 1961-62

Age	Cohort: age at survey 30-34 Observed (1)	fitted (2)	difference (1-2)	35-39 Observed (1)	fitted (2)	difference (1-2)
15-19	.25500	.31925	-.06425	.32400	.32383	-.00017
20-24	1.36000	1.19429	.16571	1.31500	1.29307	.02193
25-29	1.66700	1.66524	.00176	1.57600	1.62523	-.04923
30-34	1.69100	1.78925	-.09825	1.44200	1.50547	-.06347
35-39				1.31000	1.21700	.09300

Age	Cohort: age at survey 40-44 Observed (1)	fitted (2)	difference (1-2)
15-19	.34200	.70768	-.36568
20-24	1.42300	1.20076	.22224
25-29	1.40700	1.20408	.20292
30-34	1.36500	1.12851	.23649
35-39	1.05500	1.04049	.01451
40-44	.64700	.88052	-.23352

in the middle of the childbearing period experienced. The cohort aged 35-39 does not conform to this pattern, though the size of the deviations is small. Rather, the results for this cohort suggest that there is over-reporting in the most recent period and in the period 15-19 years ago with under-reporting in the intervening periods.

Examination of the fitted rates for the three cohorts shows rather too much variation between cohorts. In particular, the fit obtained for the cohort aged 40-44 is much flatter than for younger cohorts and has a very low peak. This fit also suggests that there are very large reporting errors in the data, though this is not apparent from the fits for younger cohorts. It is probable that the cohort suffers from omissions, as do older cohorts, and that differential omission rates over time have seriously distorted the data. In addition, the fitted rates for cohort 30-34 have a very high peak at the late age 30-34. This is implausible and it seems that the model has redistributed reported fertility in the opposite direction to that expected from the rest of the data, probably the result of insufficient information provided by this cohort.

Graduated rates are shown in Table 6.21. As for Bangladesh, the pattern parameters obtained for the cohort aged 30-34 have been rejected as unlikely. For this cohort and the two younger ones, therefore, graduated

Table 6.21: Graduated fertility rates by period; West New Guinea, 1961-62

Cohort: age at survey	Period: years before survey 0-4	5-9	10-14	15-19	20-24	25-29	30-34
15-19	.19800	.00100					
20-24	1.35600	.19800	.00100				
25-29	1.70211	1.35423	.33915	.00251			
30-34	1.57616	1.70152	1.35378	.33903	.00251		
35-39	1.21700	1.50547	1.62523	1.29307	.32383	.00240	
40-44	.88052	1.04049	1.12851	1.20408	1.20076	.70768	.07696
45-49	.26119	.80075	1.21700	1.50547	1.62523	1.29307	.32623
	Cumulated within period						
15-19	.19800	.19900	.34015	.34154	.32634	.71008	.40319
20-24	1.55400	1.55323	1.69393	1.63461	1.52710	2.00315	
25-29	3.25611	3.25475	3.31916	2.83869	3.15233		
30-34	4.83227	4.76022	4.44767	4.34416			
35-39	6.04927	5.80071	5.66467				
40-44	6.92979	6.60146					
45-49	7.19098						
TF	7.21	7.00					

rates have been calculated using the pattern parameters for the cohort aged 35-39 and reported mean parity at the survey. For the cohort aged 15-19, reported rates are used. The fitted fertility rates for cohorts aged 35-39 and 40-44 are used directly (though it must be recognised that the fit obtained for the latter is rather unsatisfactory). For the oldest cohort, fitted rates for the cohort aged 35-39 are used because of their greater plausibility.

Total fertility estimates for the two most recent periods were obtained by fitting the model to these data. Results are given in Appendix 6.2. The estimates indicate a current level of 7.2 with an increase over the past 5 years of 0.2, and previous periods suggest that the increase in fertility extends 15 years into the past. These results suggest a very slightly higher current level of fertility than that reported, but at the same time suggest that the increase in fertility is only one quarter of that reported with a rise of about 0.5 over the past 15 years.

The effect of a more peaked pattern of fertility for cohort 40-44 can be examined by considering the fits to the graduated rates in the same way that possible changes due to changing nuptiality were examined for previous data. For the most recent period, fertility at 40-44 would be reduced thereby reducing the total fertility estimate. This would increase the estimate of β, thus decreasing total fertility further. For the preceding period, fertility at age 35-39 would be increased slightly by an assumption of a more peaked pattern for cohort 40-44, thus increasing total fertility by the same amount. The estimate of β would be reduced as a result of an increase in fertility at late ages such that total fertility would be further increased. The total effect of these adjustments would be to reduce, if not remove, the reported fertility increase. It must be noted, however, that the adjustments would result in a greater difference in β for the two periods. This is probably due to the very great errors that are known to exist in these data, and can be regarded as indicative of such unreliability. The results for these data must thus be viewed with limited confidence.

CHAPTER 7

CONCLUSIONS

The transformed Gompertz model developed in earlier chapters has been shown in Chapter 6 to go some way towards correcting the errors and biases resulting from timing errors in the data. The extent to which this is true is related, not surprisingly, to the quality of the data in that there is greater residual error for poorer quality data. This can be seen from the graduated rates themselves, but has also been shown in the fits obtained for the graduated rates in the two most recent five year periods, where the magnitude of the differences in the pattern parameters can be used as an indication of the plausibility of the results.

Given the quality of the data analysed, the results obtained are generally good. It must be remembered that the sample sizes of the data for Bangladesh and Sri Lanka are not as large as would ideally be required for their analysis. The division of the data into cohorts and time periods results in the rapid reduction of cell sizes, especially where the population has already been divided into groups, as in Sri Lanka on the basis of education. Sampling errors for these data are therefore not as negligible as might be desired in the kind of approach used in fitting the model. Systematic errors, for which the approach is appropriate, are also considerable in the data for Bangladesh and West New Guinea. This type of error affects the older cohorts more than the younger cohorts, and is rendered all the more important by the fact that the analysis is based largely on older cohorts because of the small number of datapoints available for the more accurate younger cohorts.

The experience gained from these results would suggest that the model in conjunction with the fitting procedure adopted is not rigid enough to stand up to massive errors in the data. This is particularly noticeable where there are serious omissions at differential rates over time which confound the problem of misreporting.

Possible improvements in the method of fitting centre around the pattern parameters of the model. Some of the fertility patterns implied by the fitted α, β combinations are clearly implausible and differ considerably from patterns for adjacent cohorts. This occurs more often for poorer quality data where the timing errors lead to implausible patterns whilst the estimated level is largely unaffected. This relative stability of the level parameter arises from the compensatory effect of α which in response to wildly deviating values of β (from $\beta = 1$), serves to moderate F. Improvements might thus be best concerned with the moderation of either α or β, or even of both simultaneously.

The most stringent restriction on β would be to hold β constant at a value $\beta = 1$, thereby reducing the number of parameters to two and imposing the same pace of fertility as that in the standard. A slightly less stringent restriction might be to hold β constant at a value more appropriate for the data, possibly by choosing a β value from those obtained by the methods used in Chapter 6. For populations where β is clearly not constant, more flexibility could be achieved by allowing a linear trend in β over time. This would accommodate changing patterns of fertility whilst avoiding the erratic changes in β that may accompany them. Further flexibility would be afforded by merely constraining β to a bounded interval. Generally an interval of about 0.2 within the range 0.8 to 1.2 might be appropriate, but more specific intervals for particular populations might be envisaged.

Parallel restrictions to those discussed for β might be imposed on α. The most stringent is $\alpha = 0$, implying the same proportion, e^{-1}, of fertility achieved by age 25 as in the standard. More appropriate fixed values of α might be obtained as for β by choosing from those obtained by the methods used in Chapter 6. Alternatively, since α is more easily interpretable in demographic terms, it might be possible to estimate P, the proportion of fertility achieved by age 25, from the data.

Again, linear trends in α (or in P) might be introduced to allow for changing fertility patterns, and further flexibility might be gained by generally allowing α to vary by about 0.1 within the range -0.25 to +0.25, though more specific intervals would be required for some populations.

Perhaps the most obvious and rewarding improvement in the fitting procedure would be the use of single year values. As long as sample sizes are large, these would afford much greater precision than do five year age groups, and would also extend fitting to much younger cohorts, thereby utilising the more accurate reports of younger women and avoiding the need to assume patterns of fertility for these women. The results have shown that even the cohort aged 30-34 does not provide sufficient information for the reliable estimation of the parameters, and if omissions are serious the results for the oldest cohorts are also impaired.

The problem of omissions as such, in that mean parities at the time of the survey are under-reported, is not dealt with by the model. However, after graduation the problem still remains and affects the period rates and the estimated total fertilities. It might be desirable to inflate the level of fertility for cohorts affected by omissions, possibly by using the model to fit to reported mean parities up to age 35. Before such an exercise would be of value, however, improved methods of fitting are required because of the sensitivity at present to possible changes in fertility incorporated into period rates, as is evident from the fits obtained to the graduated period rates.

With the advent of improved fitting procedures, some of the approximations involved might also be re-examined. In particular, the need for the assumption that ages at mean parities are the same as those in the standard might be avoided by taking account of these age differences, based as they are on α and β, in the fitting procedure by using the value of the standard at the exact ages of the reported mean parities.

All of these suggestions for improvements in the fitting technique point to the need for a great deal more suitable data. Until such data are available, and more experience can be gained on which to base judgements, the optimum method of fitting the model must remain undeterminable.

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APPENDIX 2.1

SOME PROPERTIES OF THE MODE OF THE FIRST DERIVATIVE OF THE GOMPERTZ FUNCTION

Cumulative fertility is described by the Gompertz function

$$F(x) = FA^{B^{x-x_o}}$$

where x_o is an arbitrary origin of the age scale, and F, A and B are parameters. The first derivative of F(x) with respect to x describes age specific fertility,

$$f(x) = \frac{dF(x)}{dx} = \frac{dFA^{B^{x-x_o}}}{dx}$$

Let $v = B^{x-x_o}$, so that

$$\ln v = (x-x_o)\ln B \quad \text{and} \quad \frac{1}{v}\frac{dv}{dx} = \ln B$$

Then $\frac{dF(x)}{dx} = \frac{dF(x)}{dv}\,\frac{dv}{dx}$ and $F(x) = FA^v$

Hence $\ln F(x) = \ln F + v\ln A$

and $\frac{1}{F(x)}\,\frac{dF(x)}{dv} = \ln A$

Hence age specific fertility is described by

$$f(x) = F(x) \ \mathrm{ln}A \ v \ \mathrm{ln}B$$

$$= F \ \mathrm{ln}A \ \mathrm{ln}B \ B^{x-x_o} \ A^{B^{x-x_o}} \qquad \text{A2.1.1}$$

The mode of age specific fertility occurs at the age, x_m, at which f(x) is a maximum, that is when

$$\frac{df(x)}{dx} = 0 \quad \text{and} \quad \frac{d^2f(x)}{dx^2} < 0$$

Now

$$\frac{df(x)}{dx} = \frac{d}{dx} F \ \mathrm{ln}A \ \mathrm{ln}B \ B^{x-x_o} \ A^{B^{x-x_o}}$$

$$= F \ \mathrm{ln}A \ \mathrm{ln}B \ [\mathrm{ln}B \ B^{x-x_o} \ A^{B^{x-x_o}} + B^{x-x_o} \ \mathrm{ln}A \ \mathrm{ln}B \ B^{x-x_o} \ A^{B^{x-x_o}}]$$

$$= F \ \mathrm{ln}A \ \mathrm{ln}B \ [\mathrm{ln}B \ B^{x-x_o} \ A^{B^{x-x_o}} \ (1 + \mathrm{ln}A \ B^{x-x_o})] \qquad \text{A2.1.2}$$

Since $0 < A, B < 1$ and $F > 0$, the sign of $df(x)/dx$ is determined by (and is the opposite of) the sign of $(1 + \mathrm{ln}A \ B^{x-x_o})$, and at the maximum $1 + \mathrm{ln} \ A.B^{x_m-x_o} = 0$. Hence

$$\mathrm{ln}A.B^{x_m-x_o} = \mathrm{ln}A^{B^{x_m-x_o}} = -1$$

$$\text{and} \quad A^{B^{x_m-x_o}} = e^{-1} \qquad \text{A2.1.3}$$

Proof that this is a maximum (rather than a minimum) is given below by showing that $d^2f(x)/dx^2 < 0$. Differentiating equation A2.1.2 gives

$$\frac{d^2f(x)}{dx^2} = F\ \ln A.\ln B.\ln B[(A^{B^{x-x_o}}\ \ln B.B^{x-x_o} +$$

$$B^{x-x_o}\ \ln A.\ln B.B^{x-x_o}\ A^{B^{x-x_o}})(1 + \ln A.B^{x-x_o})$$

$$+ A^{B^{x-x_o}}\ B^{x-x_o}\ (\ln A.\ln B.B^{x-x_o})]$$

$$= F\ \ln A(\ln B)^3\ B^{x-x_o}\ A^{B^{x-x_o}}\ [(1 + \ln A.B^{x-x_o})^2 + \ln A.B^{x-x_o}]$$

The sign of this whole expression is the same as the sign of the part in square brackets. Noting that $\ln A.B^{x-x_o} = \ln A^{B^{x-x_o}}$ and using the result in A2.1.3 for $x = x_m$ shows that this is negative. Hence $f(x_m)$ is the maximum and x_m is the modal age.

Result A2.1.3 thus means that by the age of maximum fertility, e^{-1} or .368 of total fertility has been achieved, irrespective of the parameter values, A and B.

Using this result, the modal value of f(x) can be derived. From equation A2.1.1,

$$f(x_m) = F.\ln A.\ln B.B^{x_m-x_o}\ A^{B^{x_m-x_o}}$$

$$= F\ \ln A.\ln B.B^{x_m-x_o}\ e^{-1}$$

Again noting that $\ln A.B^{x_m-x_o} = -1$, gives

$$f(x_m) = -F.\ln B.e^{-1} \qquad \text{A2.1.4}$$

The maximum rate of age specific fertility is thus shown to be independent of the location parameter A.

The age of maximum fertility, x_m, in relation to the origin, x_o, is derived from equation A2.1.3. Taking double logarithms gives

$$\ln(-\ln A) + (x_m - x_o) \ln B = \ln(-\ln e^{-1}) = 0$$

$$\therefore x_m - x_o = \frac{-\ln(-\ln A)}{\ln B} \qquad \text{A2.1.5}$$

It is seen from A2.1.5 that if

$$x_m > x_o, \frac{-\ln(-\ln A)}{\ln B} > 0$$

which implies that $-\ln(-\ln A) < 0$ and $A < e^{-1}$. Similarly, $x_m < x_o$ implies $A > e^{-1}$, and $x_m = x_o$ implies $A = e^{-1}$. The size of the parameter A relative to e^{-1} thus indicates the position of the modal age in relation to the origin, as expected from its definition. The exact distance of the mode from the origin depends on both A and B.

APPENDIX 2.2

THE RELATIONSHIP BETWEEN THE GOMPERTZ PARAMETER, B, AND THE VARIANCE OF THE FIRST DERIVATIVE OF THE GOMPERTZ FUNCTION

The Gompertz function is described in Chapter 2, and its first derivative is given in Appendix 2.1. Of the three parameters describing these functions (F, A and B), it is shown here (following Murphy and Nagnur, 1972) that B is related to the variance of the first derivative, f(x) where

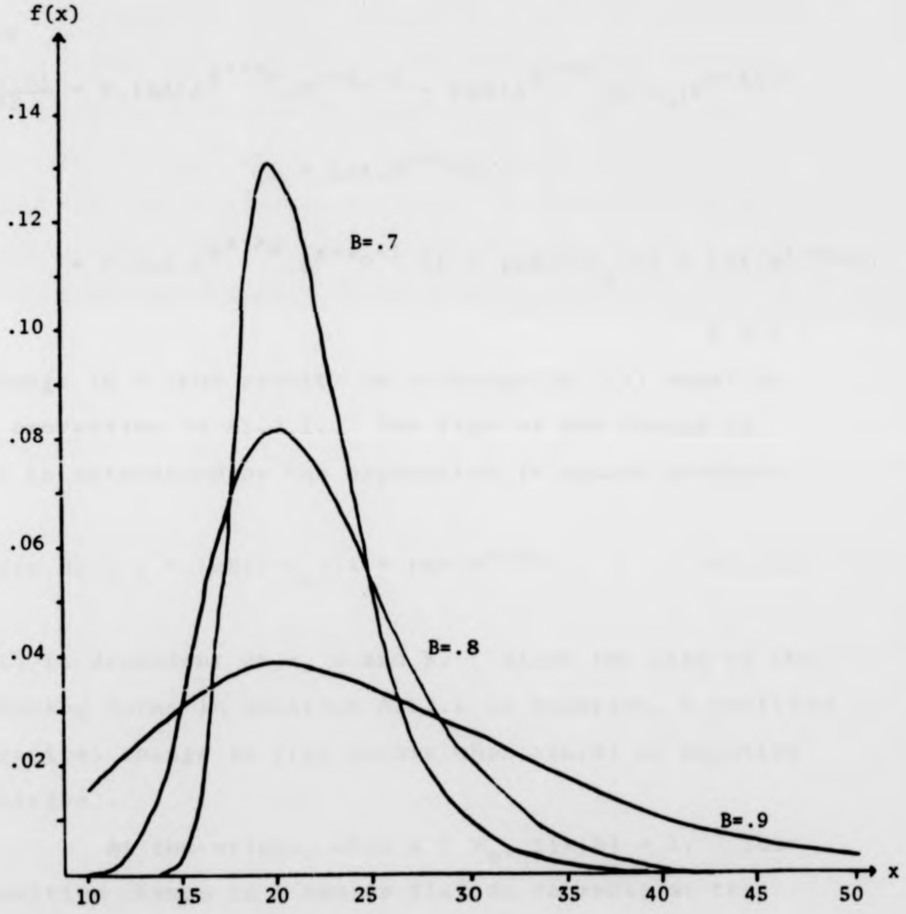
$$f(x) = \frac{dF(x)}{dx} = F\ \ln A . \ln B . B^{x-x_0}\ A^{B^{x-x_0}}$$

The effect of changing B, for fixed values of A and F, is shown in Figure A 2.2.1. As B decreases, the distribution becomes more concentrated around its mode. (The fact that the mode moves slightly as B changes for fixed A is shown in Appendix 2.1.) Algebraically, this effect is shown by taking the partial derivative of f(x) with respect to B:

$$\frac{\partial f(x)}{\partial B} = F\ \ln A\ \frac{\partial}{\partial B}\ (\ln B . A^{B^{x-x_0}} . B^{x-x_0})$$

$$= F\ \ln A[A^{B^{x-x_0}}\ B^{x-x_0}\ \frac{1}{B} + \ln B . \frac{\partial}{\partial B}(A^{B^{x-x_0}} . B^{x-x_0})]$$

Figure A2.2.1: The effect of different levels of B for F=1, A=.35, x_0=20 on age specific fertility.



where $\frac{\partial}{\partial B} A^{B^{x-x_o}}.B^{x-x_o} = B^{x-x_o}.\ln A(x-x_o)A^{B^{x-x_o}}.B^{x-x_o-1}$

$$+ A^{B^{x-x_o}} (x-x_o)B^{x-x_o-1}$$

Hence

$$\frac{\partial f(x)}{\partial B} = F.\ln A[A^{B^{x-x_o}}.B^{x-x_o-1} + \ln B(A^{B^{x-x_o}}(x-x_o)B^{x-x_o-1}$$

$$(1 + \ln A.B^{x-x_o}))]$$

$$= F.\ln A\ A^{B^{x-x_o}}.B^{x-x_o-1}\ [1 + \ln B(x-x_o)(1 + \ln A.B^{x-x_o})]$$

A 2.2.1

A change in B thus results in a change in f(x) equal to the expression in A2.2.1. The sign of the change in f(x) is determined by the expression in square brackets:

$$z(x,B) = 1 + \ln B(x-x_o)(1 + \ln A.B^{x-x_o}) \qquad \text{A2.2.2}$$

which is dependent on x, A and B. Since the sign of the remaining terms in equation A2.2.1 is negative, a positive (negative) change in f(x) occurs when z(x,B) is negative (positive).

At the origin, when $x = x_o$, $z(x,B) = 1$. Thus a positive change in B causes f(x) to decrease at the origin. Noting that $\ln A.B^{x-x_o} = \ln F(x)/F$ in equation A2.2.2, it is seen that as $x \to -\infty$, $\ln F(x)/F \to -\infty$ and $z(x,B) \to -\infty$

and as $x \to \infty$, $\ln F(x)/F \to 0$ and $z(x,B) \to -\infty$

Hence at the tails a positive change in B results in a positive change in f(x). Since $-\infty < x_0 < \infty$, an increase in B is shown to result in an increase in f(x) in the tails, accompanied by a decrease in f(x) towards the middle of the age range. In other words, B is associated with the variance of f(x). In practical terms, x ranges from 10 to 50 years, with $F(x)/F = 0$ at 10 and 1 at 50, and hence $10 < x_0 < 50$.

APPENDIX 2.3

THE EFFECT OF A CHANGE IN THE ORIGIN OF THE STANDARD ON THE TRANSFORMED GOMPERTZ MODEL

The origin of the standard in the transformed Gompertz model is defined as the point at which $Y_s(x) = 0$. This age is denoted x_{os} in the natural age scale and $F_s(x_{os}) = e^{-1}$. A change in x_{os} does not change the value of $F_s(x_{os})$; instead the shape of $F_s(x)$ changes so that $F_s(x_{os}) = e^{-1}$ and $Y_s(x_{os}) = 0$ are always true.

From Figure 2.1 in Chapter 2 it is seen that a change in the origin, the point at which $Y_s(x) = 0$, is equivalent to a vertical movement of the Y-curve. This does not change the shape of the Y-curve in any way, but merely changes its location along the Y-axis so that at each age the difference between the new (location) standard, $Y_n(x)$, and the original, $Y_s(x)$, is a fixed constant, d:

$$Y_n(x) - Y_s(x) = d \qquad \text{A2.3.1}$$

For positive (negative) d, the Y-curve is moved upwards (downwards) and the new origin is less (greater) than x_{os}. The actual difference in age between the new and original origins depends on both d and x_{os} for any given Y pattern.

Even though the pattern of transformed fertility remains the same, a change in the origin leads to a change in the pattern of cumulative fertility. From equation A2.3.1

$$F_n(x) = e^{-e^{-Y_n(x)}} = e^{-e^{-Y_s(x)-d}}$$

$$= [F_s(x)]^{e^{-d}}$$

In other words, the additive constant in the transformed scale becomes a constant power applied to $F_s(x)$ so that its effect depends on the size of $F_s(x)$. The changed origin thus implies the use of a new standard, which in turn implies that the model be respecified with new parameters. Replacing $Y_s(x)$ in the original model by its equivalent from equation A2.3.1 gives

$$Y(x) = \alpha + \beta(Y_n(x) - d)$$

$$= \alpha - \beta d + \beta Y_n(x)$$

As expected, a change in the origin of the standard affects the location parameter, α, but leaves β unchanged. In terms of cumulative fertility, the respecified model is

$$F(x) = F\ e^{-e^{-\alpha+\beta d-\beta Y_n(x)}}$$

$$F(x) = e^{-e^{-\alpha}.e^{\beta d}.e^{-\beta Y_n(x)}}$$

$$= F(P^{Q^{-d}})^{Q^{Y_n(x)}}$$

P is therefore replaced by $P' = P^{Q^{-d}}$ in the model, and it is P' that is the proportion of observed fertility achieved by the origin of the new standard. Q is not affected by changes in the origin.

The effect of a change in the origin can be seen in Figures 2.2 to 2.5. If it is assumed that the new standard is the solid line, diagram 2 shows the case when $d < 0$ and the new origin is greater than x_{os}, and diagram 3 shows $d > 0$ and the new origin less than x_{os}.

APPENDIX 2.4

THE RELATIONSHIP BETWEEN THE TRANSFORMED GOMPERTZ PARAMETER, Q, AND THE RELATIVE VARIANCES OF OBSERVED AND STANDARD AGE SPECIFIC FERTILITY

The relationship between Q and the relative variances of observed and standard age specific fertility has been shown graphically in Chapter 2. This relationship is shown algebraically in this appendix.

Age specific fertility, f(x), is represented in the transformed Gompertz model by

$$f(x) = \frac{dF(x)}{dx} = \frac{d}{dx}\, P^{Q^{Y_s(x)}}$$

$$= F \ln P \ln Q \; P^{Q^{Y_s(x)}} \; Q^{Y_s(x)} \; y_s(x) \qquad \text{A2.4.1}$$

where $y_s(x) = \frac{d}{dx} Y_s(x) = \frac{-d}{dx} \ln v$

where $v = -\ln F_s(x)$ and $\frac{dv}{dx} = \frac{-1}{F_s(x)} f_s(x)$

Hence $y_s(x) = \frac{-1}{-\ln F_s(x)} \cdot \frac{-1}{F_s(x)} \cdot f_s(x)$

$$= \frac{f_s(x)}{F_s(x).\ln F_s(x)}$$

and therefore equation A2.4.1 becomes

$$f(x) = \frac{-F\ f_s(x)}{F_s(x).\ln F_s(x)}\ \ln P.\ln Q.P^{Q^{Y_s(x)}}.Q^{Y_s(x)} \qquad \text{A2.4.2}$$

The effect of a change in Q is shown by taking the partial derivative of f(x) with respect to Q:

$$\frac{\partial f(x)}{\partial Q} = \frac{-F\ f_s(x)}{F_s(x)\ \ln F_s(x)}\ \ln P[\ \frac{1}{Q}\ .\ P^{Q^{Y_s(x)}}.Q^{Y_s(x)} +$$

$$\ln Q\{P^{Q^{Y_s(x)}}.Q^{Y_s(x)-1}\ Y_s(x) + Q^{Y_s(x)}\ \ln P.P^{Q^{Y_s(x)}}$$

$$.Y_s(x).Q^{Y_s(x)-1}\}]$$

$$= \frac{-F\ f_s(x)}{F_s(x)\ \ln F_s(x)}\ \ln P.P^{Q^{Y_s(x)}}.Q^{Y_s(x)-1}$$

$$[1 + Y_s(x)\ \ln Q(1 + \ln P.Q^{Y_s(x)})] \qquad \text{A2.4.3}$$

The sign of $\frac{\partial f(x)}{\partial Q}$ is the opposite of the sign of the expression in square brackets in A2.4.3, that is the sign of

$$z(x,Q) = 1 + Y_s(x)\ \ln Q(1 + \ln P.Q^{Y_s(x)}) \qquad \text{A2.4.4}$$

which is dependent on P, Q and $Y_s(x)$. The sign of $z(x,Q)$ is determined by the relative sizes of lnP and Q and on whether $Y_s(x) \gtrless 0$. If $Y_s(x) = 0$, $z(x,Q) = 1$ indicating a negative change in f(x). Since, by definition, the origin occurs in the interval $10 < x_{os} < 50$, this means that f(x) decreases in at least part of this age range when Q increases. When $Y_s(x)$ tends to its limits at 10 and 50, $z(x,Q)$ is negative such that $f(x)$ increases:

$$\text{as } x \to 10, \quad z(x,Q) \to -\infty$$
$$\text{as } x \to 50, \quad z(x,Q) \to -\infty$$

Hence at the limits of the childbearing range an increase in Q results in an increase in $f(x)$. These results indicate that as Q increases, the $f(x)$ curve is flattened at the origin of the standard and increased in the tails: as Q is increased (decreased) the variance of observed age specific fertility is increased (decreased). Since the variance of standard age specific fertility is fixed, and since $f(x)$ is a function of $f_s(x)$, Q can also be regarded as relating to the relative variances of observed and standard age specific fertility in that as Q increases (decreases), the observed variance increases (decreases) relative to the standard variance. Equality of the two variances cannot be determined from the Q parameter alone. Only if $P = e^{-1}$ and $Q = e^{-1}$ are the distributions identical, and hence the variances equal, though various combinations of P and Q could, of course, produce the same value of the variance numerically.

APPENDIX 3.1

THE ESTIMATION OF THE PARAMETERS a_0, k and m FROM OBSERVED DATA

A. Estimation of a_0 and k from nuptiality data

The estimation of the nuptiality parameters, a_0 and k, is done from knowledge of the first marriage distribution. The method is that of Coale (1971), and involves fitting a standard schedule, described by the function G(x) (see Chapter 3), to actual nuptiality data.

Let P_i denote the proportions ever married by 5 year age group i, where i = 1 for the first age group in which marriage occurs. Thus if marriage begins at an age between 10 and 15, P_1 refers to the age group 10-14; if, however, first marriage does not begin until after 15 years (but before 20), P_1 refers to the age group 15-19. Two sets of ratios, one of which can usually be calculated from the observed data, are proposed by Coale as a basis for estimating a_0 and k.

The first set is R_1, R_2 and R_3 where

$$R_i = \frac{P_i}{P_{i+1}}$$

that is the ratio of the proportions ever married in the ith age group to the next. (In calculating R_i, the proportions ever married at the midpoints of age groups are used.) Any particular value of R_i $(i = 1,2,3)$ can occur by different combinations of a_o and k, but a pair of ratios (R_1 and R_2, or R_2 and R_3) can occur only by one (a_o,k) combination. Determination of such a combination is done by locating R_i and R_{i+1} (where i = 1 or 2) in Table 1A in Coale (1971), interpolating between rows to obtain k, and between columns to obtain a_o. In fact, $a_o - a_i$, where a_i is the beginning age of the first age group in which marriage occurs, is the quantity obtained by interpolation. .If this were done for both pairs of ratios (R_1 and R_2, and R_2 and R_3), the estimates obtained for a_o and k would not be the same unless the observed nuptiality schedule were a perfect fit to a standard schedule (with parameters equal to the estimates obtained). Since this is unlikely to occur in practice, a method of choosing between the two sets of estimates is required. Coale recommends using R_1 and R_2 if $R_1 > (1 - R_3)$, and R_2 and R_3 if $R_1 < (1 - R_3)$.

The second set of ratios which can be used to estimate values of a_o and k, is RA_1, RA_2 and RA_3, calculated in the same way as R_i except that endpoint data are used rather than midpoints.

Endpoint data are the same as the average proportions ever married except for a factor of 5, which cancels out in the ratios. Hence if P_i now refers to proportions ever married by the end of the ith 5 year age group, and P_i denotes average proportions first married during the ith age group, then

$$P_i = 5 \sum_{j=1}^{i} p_j$$

$$\text{and } RA_i = \frac{P_i}{P_{i+1}} = \frac{\sum_{j=1}^{i} p_j}{\sum_{j=1}^{i+1} p_j} \quad \text{for } i = 1, 2, 3$$

Values of RA_1 and RA_2 or of RA_2 and RA_3 (the choice of which is determined in the same way as for pairs of R_i) are located in Table 2A in Coale (1971), thus arriving at estimates of a_o and k.

By way of an example, consider the following data on proportions of females currently married (which can be approximated to proportions ever married) in Ceylon, 1946 (from Lesthaeghe, 1971, Appendix):

age	10-14	15-19	20-24	25-29
P_i	.007	.259	.685	.845

The ratios R_1, R_2 and R_3 are .027, .378 and .811 respectively. Since $R_1 < 1 - R_3$, R_2 and R_3 are used to determine a_o and k from Table 1A. A value of $R_2 = .378$ is obtainable for $a_o = 3.5$, $k = .543$, the corresponding value of R_3 being .839. A second value is obtained for $a_o = 3.0$, $k = .632$ with $R_3 = .810$. Interpolation to obtain the correct value of R_3, gives $a_o = 3.02$ and $k = .629$. Since marriage begins in the age group 10-14, $a_1 = 10$ years and the final nuptiality parameter estimates are $a_o = 13.02$ years, $k = .629$.

B. Estimation of m from marital fertility data

The degree of voluntary birth control, m, in a population is estimated from the observed marital fertility schedule, r(x), using the basic equation

$$\frac{r(x)}{n(x)} = M.e^{m.v(x)} \qquad \text{A3.1.1}$$

where x is age, n(x) is natural fertility, v(x) is a standard pattern of birth control and M is a scale factor equating r(x) to n(x) for some chosen value of x. Rearranging equation A3.1.1 gives

$$m = \ln\left(\frac{r(x)}{M.n(x)}\right) / v(x) \qquad \text{A3.1.2}$$

Values of n(x) and v(x) appear in Table 3.1 in Chapter 3 and are reproduced in the example below; and M is chosen to equate r(20-24) with n(20-24) since 20-24 is the last age group before voluntary control begins. Using equation A3.1.2, values of m can be obtained for age groups 25-29 to the last age group for which data are available. If the pattern of control in the observed population were identical to the standard pattern, the value of m would be the same at all ages. Such uniformity is rarely attained: however, the average value of m serves as a convenient summary measure of the extent of voluntary control, and the variance serves as a measure of the goodness of fit.

As an example, values of m for the Swedish cohort aged 25-29 in 1896-1900 (from Knodel, 1977) are calculated below. The data are as follows:

age	20-24	25-29	30-34	35-39	40-44	45-49
r(x)	.456	.369	.283	.205	.094	.010
n(x)	.4597	.4309	.3946	.3223	.1671	.0237
v(x)	0.000	-.279	-.677	-1.042	-1.414	-1.671

with

$$M = \frac{r(20-24)}{n(20-24)} = \frac{.456}{.4597} = .99195.$$

The value of m for age 25-29 is thus

$$\ln\left(\frac{0.369}{(.99195)(.4309)}\right) / -0.279$$

$$= \ln(.86330)/-0.279$$

$$= -0.14700/-0.279$$

$$= 0.527$$

Similar calculations for ages 30-34 to 45-49 result in a series of m values:

age	25-29	30-34	35-39	40-44	45-49
m	0.53	0.48	0.43	0.40	0.52

with a mean of 0.47 and variance of 0.0032.

The above method of estimating m has been used by Knodel (1977) and is used in Chapter 4 to compare the simulated data, developed in that chapter, with Knodel's work. The method suffers from its dependence on only one age group (20-24) to calculate M, which affects the values of m. More recently, Coale and Trussell (1978) have presented a method of estimating M and m simultaneously using all reliable data points. Taking logarithms of equation A3.1.1 gives

$$\ln(r(x)/n(x)) = \ln M + m\, v(x)$$

which is linear in v(x) and can be solved by ordinary least squares.

C. Estimation of a_o, k and m from fertility data for all women

Simultaneous estimation of nuptiality and birth control parameters involves fitting observed age specific fertility to the Coale-Trussell model schedules. This can be done crudely by choosing the schedule from the published set (Coale and Trussell, 1974) that most closely resembles the pattern of observed fertility. The fitting procedure can be much improved, however, by use of an iterative minimisation procedure facilitated by the use of a computer. Such a method is outlined below.

The minimisation program MINUIT (James and Roos, 1971) is used to minimise the sum of squares of the differences between the observed and fitted age specific fertility schedules. The fitted schedule is calculated from the parameters, a_o, k and m. An initial set of parameters is provided by the user, and subsequent changes in parameter values are determined by MINUIT according to the size of the deviations of the fitted values from the observed. By way of an example, estimates of a_o, k and m for the standard fertility schedule are derived. The complete program and output are reproduced below.

```
      PROGRAM FITARM  (INPUT, OUTPUT, TAPE1 = INPUT, TAPE2 = OUTPUT)
      CALL MINUITS
      STOP
      END


      SUBROUTINE FCN (NPAR, G, F, X, IFLAG)
      DIMENSION X(3), FIVYR(8), OBS(8), DIFF(8)
      IF (IFLAG .GT. 1) GO TO 1
      READ(1,555)
  555 FORMAT (50H                                                  )
      READ (1,500) (OBS(I), I = 1,8)
  500 FORMAT (8F10.5)
      TOT=0.0
      DO 6 I=1,8
      TOT=TOT+OBS(I)
6     CONTINUE
      DO 7 I=1,8
      OBS(I)=OBS(I)/TOT
7     CONTINUE
      IN = 1
      IF (OBS(1) .LE. 0.0) IN = 2
    1 AGE = X(1)
      RATE = X(2)
      DEG = X(3)
      CALL    FERT (IFLAG, AGE, RATE, DEG, FIVYR)
      F = 0.0
      IF (IN .EQ. 2) FIVYR(2) = FIVYR(2) + FIVYR(1)
      DO 2 I = IN,8
       F=F+((FIVYR(I)-OBS(I))**2)
    2 CONTINUE
      IF (IFLAG .NE. 3) GO TO 3
      WRITE (2,501) (OBS(I), I = 1,8)
      DO 4 I = IN,8
      DIFF(I) = FIVYR(I) - OBS(I)
    4 CONTINUE
      WRITE(2,502) (DIFF(I), I = 1,8)
      WRITE (2,504) F
      TOBS = 0.0
      SOBS = 0.0
      DO 5 I = 2,8
      EYE = I - 1
      TOBS = TOBS + EYE * OBS(I)
      SOBS = SOBS + EYE * EYE * OBS(I)
    5 CONTINUE
      SOBS = SOBS - TOBS * TOBS - 1.0 / 12.0
      TOBS = 17.5 + 5.0 * TOBS
      SOBS = 5.0 * SQRT(SOBS)
      ROBS = (OBS(1) + OBS(2)) / OBS(3)
      WRITE(2,503) TOBS, SOBS, ROBS
      WRITE (2,555)
  501 FORMAT (1H0 / 1H0 / 1H0, 8(F7.5, 3X), 'OBSERVED FIVE YEAR RATIOS')
  502 FORMAT (1H0, 8(F7.5, 3X), 'DIFFERENCES')
  503 FORMAT (1H0 / 1H0 / 1H0, 'OBSERVED STATISTICS        MU        SIG'
     1 '         RHO' / 25X, 3(F6.3, 4X) ////////////)
  504 FORMAT (1H0, 'OBJECTIVE FUNCTION' F10.6)
    3 RETURN
      END
```

```
      SUBROUTINE   FERT (IFLAG, AGE, RATE, DEG, FIVYR)
      DIMENSION ASFR(40), FNAT(40), DEP(40), ZED(401), FIVYR(8),FLIST(8)
      DATA FNAT / .005, .100, .175, .225, .275, .325, .375, .421, .450,
     1            .475, .477, .475, .470, .465, .460, .455, .449, .442,
     2            .435, .428, .420, .410, .400, .389, .375, .360, .343,
     3            .325, .305, .280, .247, .207, .167, .126, .087, .055,
     4            .035, .021, .011, .003 /
      DATA DEP / 10*(0.0), .004, .03, .06, .10, .15, .20, .25, .31, .37,
     1           .44, .52, .60, .68, .76, .83, .90, .97, 1.04, 1.11,
     2           1.18, 1.25, 1.32, 1.39, 1.46, 1.53, 1.59, 1.64, 1.67,
     3           1.69, 1.70 /
      DATA MIN / 0 /
      IF (MIN .GT. 0) GO TO 20
      MIN = 1
      DO 21 M = 1,40
      DEP(M) = -DEP(M)
   21 CONTINUE
   20 MARAGE = IFIX(AGE * 10.0 - 99.0)
      DO 22 M = 1, MARAGE
      ZED(M) = 0.0
   22 CONTINUE
      ZL = 0.0
      NEXT = MARAGE + 1
      Y = HITCH(0.0, RATE)
      DO 23 M = NEXT, 401
      X = FLOAT(M) / 10.0 - AGE + 9.9
      ZU = HOP (X,RATE)
      TRAP = (ZU + ZL) / 20.0
      ZED(M) = TRAP + ZED(M-1)
      ZL = ZU
   23 CONTINUE
      BARS = 0.0
      DO 24 M = 1,40
      AVEM = 0.0
      DO 25 MUM = 1,10
      N = (M - 1) * 10 + MUM
      AVEM = AVEM + (ZED(N) + ZED(N+1)) / 20.0
   25 CONTINUE
      ASFR(M) = AVEM * FNAT(M) * EXP(DEG * DEP(M))
      BARS = BARS + ASFR(M)
   24 CONTINUE
      DO 26 M = 1,40
      ASFR(M) = ASFR(M) / BARS
   26 CONTINUE
      DO 27 K = 1,8
      BIT = 0.0
      DO 28 J = 1,5
      JAY = (K-1) * 5 + J
      BIT = BIT + ASFR(JAY)
   28 CONTINUE
      FIVYR(K) = BIT
   27 CONTINUE
      IF (IFLAG .EQ. 3) GO TO 30
      RETURN
   30 SUM = 0.0
      SSQ = 0.0
      A = 9.5
      DO 31 M = 1,40
      A = A + 1.0
      SUM = SUM + A * ASFR(M)
      SSQ = SSQ + A * A * ASFR(M)
   31 CONTINUE
      EMU = SUM
      VAR = SSQ - SUM * SUM - 1.0 / 12.0
      SIG = SQRT(VAR)
      RHO = FIVYR(2) / FIVYR(3)
      WRITE (2,600) AGE, RATE, DEG
      WRITE (2,601) EMU, SIG, RHO
      WRITE (2,602)
      DO 32 J = 1,5
      DO 33 K = 1,8
      KAY = (K-1) * 5 + J
      FLIST(K) = ASFR(KAY)
   33 CONTINUE
      WRITE (2,603) (FLIST(K), K = 1,8)
   32 CONTINUE
      WRITE (2,604) (FIVYR(K), K = 1,8)
  600 FORMAT (1H1,'AGE         RATE        DEG' / 1X, 3(F6.3, 4X) / 1X)
  601 FORMAT (1H0,'MU          SIG         RHO' / 1X, 3(F6.3, 4X) / 1X)
  602 FORMAT (1H0,'10 - 14   15 - 19   20 - 24   25 - 29   30 - 34   ',
     1           '35 - 39   40 - 44   45 - 49' / 1X)
  603 FORMAT (1X, 8(F7.5, 3X))
  604 FORMAT (1H0, 8(F7.5, 3X))
      RETURN
      END

      FUNCTION HITCH (X, RATE)
      CONS = 0.19465 / RATE
      U = -0.174 / RATE
      V = -0.2881 / RATE
      W = 6.06 * RATE
      HITCH = 0.0
      RETURN
      ENTRY HOP
      HITCH = CONS * EXP(U * (X-W) - EXP(V * (X-W)))
      RETURN
      END
```

```
AGE       RATE      DEG
12.517     .457      .316

MU        SIG       RHO
28.310     7.286     .542

10 - 14   15 - 19   20 - 24   25 - 29   30 - 34   35 - 39

0.00000   .00005    .04663    .04827    .04109    .03133
0.00000   .01729    .04867    .04724    .03915    .02920
 .00001   .02743    .04949    .04588    .03727    .02707
 .00043   .03831    .04951    .04444    .03536    .02485
 .00291   .04274    .04683    .04287    .03336    .02232

 .00341   .13200    .24334    .22869    .19624    .13477


 .00277   .13307    .24147    .23130    .19757    .13401

 .00064  -.00107    .00187   -.00261   -.00133    .00076

OBJECTIVE FUNCTION  .000016


OBSERVED STATISTICS     MU        SIG       RHO
                      29.296     7.265      .563




 STANDARD
```

```
40 - 44   45 - 49

 .01926    .00385
 .01579    .00241
 .01246    .00143
 .00919    .00075
 .00621    .00020

 .06290    .00865


 .06169    .00812   OBSERVED FIVE YEAR RATIOS
 .00121    .00053   DIFFERENCES
```

```
                                   ************************
                                   *    D506    MINUITS   *
                                   *    VERSION 11.76     *
                                   * DATA BLOCK NO.    1  *
**********************************************************************************************************
   STANDARD                                                                    TIME  2.962
**********************************************************************************************************
**********************************************************************************************************
     1          AGE    12.0000       1.00000          10.0000        20.0000
     2          RATE   .500000       .100000          .500000E-01    5.00000
     3          DEG    .500000       .100000         0.              5.00000
**********************************************************************************************************

FIRST ENTRY TO FCN

   FCN VALUE        CALLS      TIME      EDM        INT.EXT. PARAMETER        VALUE          ERROR          INTERN.VALUE    INT.STEP SIZE
 .76457296E-03        1        3.04     0.            1    1          AGE    .12000E+02     .10000E+01     -.54350E+00     .25779E+00
                                                      2    2          RATE   .50000E+00     .10000E+00     -.35924E+00     .70591E-01
                                                      3    3          DEG    .50000E+00     .10000E+00     -.92730E+00     .64435E-01

***********
***   1****
***********
***********
***   2*****MINIMIZE
***********
START SIMPLEX MINIMIZATION                            CONVERGENCE CRITERION -- ESTIMATED DISTANCE TO MINIMUM (EDM) .LT.    .10E+00

   FCN VALUE        CALLS      TIME      EDM        INT.EXT. PARAMETER        VALUE          ERROR          INTERN.VALUE    INT.STEP SIZE
 .44404640E-04       15        3.43     .71E-03       1    1          AGE    .12698E+02     .86256E+01     -.67845E+00     .16505E+00
                                                      2    2          RATE   .46058E+00     .93148E+00     -.98652E+00     .28276E-01
                                                      3    3          DEG    .35102E+00     .44110E+01     -.10345E+01     .10714E+00

SIMPLEX MINIMIZATION HAS CONVERGED

   FCN VALUE        CALLS      TIME      EDM        INT.EXT. PARAMETER        VALUE          ERROR          INTERN.VALUE    INT.STEP SIZE
 .44404640E-04       16        3.47     .71E-02       1    1          AGE    .12698E+02     .86256E+01     -.67845E+00     .16505E+00
                                                      2    2          RATE   .46058E+00     .93148E+00     -.98652E+00     .28276E-01
                                                      3    3          DEG    .35102E+00     .44110E+01     -.10345E+01     .10714E+00

                                                    ERRORS CORRESPOND TO FUNCTION CHANGE OF      1.0000
START MIGRAD MINIMIZATION.                          CONVERGENCE CRITERIA  --  ESTIMATED DISTANCE TO MINIMUM (EDM) .LT.   .10E-05
                                                    OR EDM .LT.   .10E+00 AND FRACTIONAL CHANGE IN VARIANCE MATRIX .LT.   .40E-01

   FCN VALUE        CALLS      TIME      EDM        INT.EXT. PARAMETER        VALUE          ERROR          INTERN.VALUE    INT.STEP SIZE
 .44404640E-04       22        3.66     .59E-04       1    1          AGE    .12698E+02     .63481E+01     -.67845E+00     .87462E-01
                                                      2    2          RATE   .46058E+00     .37023E+01     -.98652E+00     .15014E-01
                                                      3    3          DEG    .35102E+00     .38670E+01     -.10345E+01     .56404E-01

MIGRAD MINIMIZATION HAS CONVERGED

   FCN VALUE        CALLS      TIME      EDM        INT.EXT. PARAMETER        VALUE          ERROR          INTERN.VALUE    INT.STEP SIZE
 .15951496E-04       43        4.27     .35E-05       1    1          AGE    .12537E+02     .62561E+01     -.51500E+00     -.52981E-02
                                                      2    2          RATE   .45718E+00     .35417E+01     -.98401E+00     .76194E-03
                                                      3    3          DEG    .31571E+00     .30470E+01     -.10628E+01     -.18597E-02

                                                    ERRORS CORRESPOND TO FUNCTION CHANGE OF      1.0000

             INTERNAL COVARIANCE MATRIX             LAST FRACTIONAL CHANGE WAS    .011919
             CORRELATION COEFFICIENTS
         INT.    1      2
           2 -.286
           3  .417   .122


                                                                     GLOBAL CORRELATION
                                                         PARAMETER      COEFFICIENT
                                                   1           AGE         .28880
                                                   2           RATE        .15223
                                                   3           DEG         .23719

***********
***   3****EXIT
***********

CALL TO FCN WITH IFLAG = 3
```

APPENDIX 3.2

MODIFICATIONS TO THE COALE-TRUSSELL MODEL COMPUTER PROGRAM

The computer program used to generate the Coale-Trussell set of model fertility schedules is reproduced in Appendix A of Coale and Trussell (1974). A modified version was available at the start of this project and was further modified as described below.

The only modification of significance is the extension of the model to ages 10 to 12.5. This involved the expansion of arrays and the provision of values of natural fertility at these early ages. These values were $n(10.5) = .005$ and $n(11.5) = .100$, determined to maintain a smooth $n(x)$ curve. Since these values are small, and since the numbers married at these young ages are small, quite large relative deviations from the chosen values produce very little effect on fertility.

Other modifications pertain only to the form of output. The program is reproduced below.

```
      PROGRAM REGS(TAPE1,INPUT,OUTPUT,TAPE7)
C
      DIMENSION FS(8)
      DIMENSION JFS(8)
      COMMON NRUN,NUM,IPRINT
C
      PRINT 96
   95 FORMAT(1H1)
      READ 100,NRUN
  100 FORMAT(I3)
      NUM=0
C                   IF  IPRINT=0 NO PRINTING
C                   IF  IPRINT=1    PRINTING
      IPRINT=0
   1  CONTINUE
      IF(NUM.EQ.NRUN) GO TO 5
      READ 99,AGE,PAKMIN,PAKMAX,PAKINT,DEGMIN,DEGMAX,DEGINT
   99 FORMAT(F5.1,6F5.2)
C
      CALL EFSKED(AGE,PAK,DEG,EMU,SIG,RHO,SKU,PAR0,PAR1,PAR2,FS)
      PLOOP=0.0
    3 CONTINUE
      PAK=PAKMIN+PAKINT*PLOOP
      IF(PAK.GT.PAKMAX) GO TO 1
      SMAM=AGE+11.37*PAK
C
      CALL NUPSET(AGE,PAK,DEG,EMU,SIG,RHO ,SKU,PAR0,PAR1,PAR2,FS)
      DLOOP=0.0
    4 CONTINUE
      DEG=DEGMIN+DEGINT*DLOOP
      IF(DEG.GT.DEGMAX) GO TO 2
C
      CALL FERTIL(AGE,PAK,DEG,EMU,SIG,RHO,SKU,PAR0,PAR1,PAR2,FS)
C
C                   INCLUDE HERE CARDS TO RESTRICT AREA OF INTEREST
      NUM=NUM+1
C
C
C                   RESTRICT PRINTING HERE IF IPRINT=0
      IF(IPRINT) 200,200,201
  201 CONTINUE
      PRINT 98, AGE,EMU,PAR0,RHO,PAK,SIG,PAR1,SMAM,DEG,SKU,PAR2
   98 FORMAT(////1H0,≠AGE=≠,F4.1,5X,≠MEAN=≠,F5.2,5X,≠P0/P1=≠,F6.4,5X,
     1  ≠ RHO=≠,F5.2/
     2  1H ,≠  K=≠,F5.2,4X,≠  SD=≠,F5.2,5X,≠P1/P2=≠,F6.4,5X,≠SMAM=≠,F5.2
     3  /
     4  1H ,≠  M=≠,F5.2,4X,≠SKEW=≠,F6.3,4X,≠P2/P3=≠,F6.4)
  200 CONTINUE
      JAGE=AGE*10
      JPAK=PAK*10
      JDEG=DEG*10
      JEMU=EMU*100+0.5
      JSIG=SIG*100+0.5
      JSKU=SKU*1000+0.5
      JPAR0=PAR0*10000+0.5
      JPAR1=PAR1*10000+0.5
      JPAR2=PAR2*10000+0.5
      JRHO=RHO*1000+0.5
      JSMAM=SMAM*100+0.5
      DO 301 I=1,8
      JFS(I)=FS(I)*100000+0.5
  301 CONTINUE
      WRITE(7,300)  JAGE,JPAK,JDEG,JEMU,JSIG,JSKU,JPAR0,JPAR1,JPAR2,
     1   JRHO,JSMAM,(JFS(I),I=1,8)
  300 FORMAT(I3,2I2,2I1,2X,I4,2I3,3I4,I3,I4,1X,8I5)
      CALL CTFERT(15,AGE,PAK,DEG)
C
   10 CONTINUE
C
      DLOOP=DLOOP+1.0
      GO TO 4
    2 CONTINUE
      PLOOP=PLOOP+1.0
      GO TO 3
C
    5 STOP
      END
```

```
      SUBROUTINE EFSKED(AGE,RATE,DEG,EMU,SIG,RHO,SKU,PAR0,PAR1,PAR2,FS)
C
C
      DIMENSION ASFR(40), FNAT(40), DEP(40), ZED(401)
      DIMENSION G(4),FS(8)
      DIMENSION AVEM(40)
      COMMON NRUN,NUM,IPRINT
C
      DATA MINUS / 0 /
C
      DATA FNAT / .005, .100, .175, .225, .275, .325, .375, .421, .460,
     1            .475, .477, .475, .470, .465, .460, .455, .449, .442,
     2            .435, .428, .420, .410, .400, .389, .375, .360, .343,
     3            .325, .305, .280, .247, .207, .167, .126, .087, .055,
     4            .035, .021, .011, .003 /
C
      DATA DEP / 10*(0.0), .004, .03, .06, .10, .15, .20, .25, .31, .37,
     1           .44, .52, .60, .68, .76, .83, .90, .97, 1.04, 1.11,
     2           1.18, 1.25, 1.32, 1.39, 1.46, 1.53, 1.59, 1.64, 1.67,
     3           1.69, 1.70 /
C
      IF (MINUS .GT. 0) GO TO 20
      DO 21 M = 1,40
      DEP(M) = -DEP(M)
   21 CONTINUE
      MINUS = 1
C
   20 MARAGE = IFIX(AGE * 10.0 - 99.0)
      DO 22 M = 1, MARAGE
      ZED(M) = 0.0
   22 CONTINUE
C
      RETURN
C
      ENTRY NUPSET
C
      X = 0.0
      ZL = HITCH (X,RATE)
      NEXT = MARAGE + 1
C
      DO 23 M = NEXT, 401
      X = FLOAT(M) / 10.0 - AGE + 9.9
      ZU = HCP (X,RATE)
      TRAP = (ZU + ZL) / 20.0
      ZED(M) = TRAP + ZED(M-1)
      ZL = ZU
   23 CONTINUE
C
      DO 24 M = 1,40
      AVEM(M)=0.0
C
      DO 25 MUM = 1,10
      N = (M - 1) * 10 + MUM
      AVEM(M)=AVEM(M)+(ZED(N) + ZED(N+1))/20.0
   25 CONTINUE
  24  CONTINUE
C
      RETURN
C
      ENTRY FERTIL
C
      BABS = 0.0
      DO 29 M=1,40
      ASFR(M)=AVEM(M)* FNAT(M) * EXP(DEG * DEP(M))
      BABS = BABS + ASFR(M)
  29  CONTINUE
```

```
C
      SUM = 0.0
      SSQ = 0.0
      A = 9.5
C
      DO 26 M = 1,40
      ASFR(M) = ASFR(M) / BARS
      A = A + 1.0
      SUM = SUM + A * ASFR(M)
      SSQ = SSQ + A * A * ASFR(M)
   26 CONTINUE
C
C     IF(IPRINT) 300,300,301
 301  CONTINUE
      EMU = SUM
      VAR = SSQ - SUM * SUM - 1.0 / 12.0
      SIG = SQRT(VAR)
C
      TOP = 0.0
      DO 27 M = 6,10
      TOP = TOP + ASFR(M)
   27 CONTINUE
C
      BOT = 0.0
      DO 28 M = 11,15
      BOT = BOT + ASFR(M)
   28 CONTINUE
C
      RHO = TOP / BOT
C
      Z=0.0
      A=9.5
      DO 30 I=1,40
      A=A+1.0
      Z=Z+(A-EMU)**3*ASFR(I)
   30 CONTINUE
      SKU=Z/SIG**3
C
  300 CONTINUE
      DO 201 J=1,8
      FS(J)=0.0
      L=5*(J-1)
      DO 202 K=1,5
      FS(J)=FS(J)+ASFR(L+K)
  202 CONTINUE
  201 CONTINUE
C     IF(IPRINT) 400,400,401
  401 CONTINUE

      DO 200 J=1,4
      L=5*(J-1)
      Q(J)=(4.5*ASFR(L+1) + 3.5*ASFR(L+2) + 2.5*ASFR(L+3) + 1.5*ASFR(L+4
     1  ) + 0.5*ASFR(L+5))/5.0
  200 CONTINUE
      Q1=Q(1)
      Q2=Q(2)+5.0*FS(1)
      Q3=Q(3)+5.0*(FS(1)+FS(2))
      Q4=Q(4)+5.0*(FS(1)+FS(2)+FS(3))
      PAR0=Q1/Q2
      PAR1=Q2/Q3
      PAR2=Q3/Q4
  400 CONTINUE
      RETURN
      END
```

```
C
      SUM = 0.0
      SSQ = 0.0
      A = 9.5
C
      DO 26 M = 1,40
      ASFR(M) = ASFR(M) / BARS
      A = A + 1.0
      SUM = SUM + A * ASFR(M)
      SSQ = SSQ + A * A * ASFR(M)
   26 CONTINUE
C
C     IF(IPRINT) 300,300,301
  301 CONTINUE
      EMU = SUM
      VAR = SSQ - SUM * SUM - 1.0 / 12.0
      SIG = SQRT(VAR)
C
      TOP = 0.0
      DO 27 M = 6,10
      TOP = TOP + ASFR(M)
   27 CONTINUE
C
      BOT = 0.0
      DO 28 M = 11,15
      BOT = BOT + ASFR(M)
   28 CONTINUE
C
      RHO = TOP / BOT
C
      Z=0.0
      A=9.5
      DO 30 I=1,40
      A=A+1.0
      Z=Z+(A-EMU)**3*ASFR(I)
   30 CONTINUE
      SKU=Z/SIG**3
C
  300 CONTINUE
      DO 201 J=1,8
      FS(J)=0.0
      L=5*(J-1)
      DO 202 K=1,5
      FS(J)=FS(J)+ASFR(L+K)
  202 CONTINUE
  201 CONTINUE
C     IF(IPRINT) 400,400,401
  401 CONTINUE
 
      DO 200 J=1,4
      L=5*(J-1)
      Q(J)=(4.5*ASFR(L+1) + 3.5*ASFR(L+2) + 2.5*ASFR(L+3) + 1.5*ASFR(L+4
     1  ) + 0.5*ASFR(L+5))/5.0
  200 CONTINUE
      Q1=Q(1)
      Q2=Q(2)+5.0*FS(1)
      Q3=Q(3)+5.0*(FS(1)+FS(2))
      Q4=Q(4)+5.0*(FS(1)+FS(2)+FS(3))
      PAR0=Q1/Q2
      PAR1=Q2/Q3
      PAR2=Q3/Q4
  400 CONTINUE
      RETURN
      END
```

```
C
      SUM = 0.0
      SSQ = 0.0
      A = 9.5
C
      DO 26 M = 1,40
      ASFR(M) = ASFR(M) / BARS
      A = A + 1.0
      SUM = SUM + A * ASFR(M)
      SSQ = SSQ + A * A * ASFR(M)
   26 CONTINUE
C
C     IF(IPRINT) 300,300,301
 301  CONTINUE
      EMU = SUM
      VAR = SSQ - SUM * SUM - 1.0 / 12.0
      SIG = SQRT(VAR)
C
      TOP = 0.0
      DO 27 M = 6,10
      TOP = TOP + ASFR(M)
   27 CONTINUE
C
      BOT = 0.0
      DO 28 M = 11,15
      BOT = BOT + ASFR(M)
   28 CONTINUE
C
      RHO = TOP / BOT
C
      Z=0.0
      A=9.5
      DO 30 I=1,40
      A=A+1.0
      Z=Z+(A-EMU)**3*ASFR(I)
  30  CONTINUE
      SKU=Z/SIG**3
C
  300 CONTINUE
      DO 201 J=1,8
      F5(J)=0.0
      L=5*(J-1)
      DO 202 K=1,5
      F5(J)=F5(J)+ASFR(L+K)
  202 CONTINUE
  201 CONTINUE
C     IF(IPRINT) 400,400,401
  401 CONTINUE

      DO 200 J=1,4
      L=5*(J-1)
      Q(J)=(4.5*ASFR(L+1) + 3.5*ASFR(L+2) + 2.5*ASFR(L+3) + 1.5*ASFR(L+4
     1  ) + 0.5*ASFR(L+5))/5.0
  200 CONTINUE
      Q1=Q(1)
      Q2=Q(2)+5.0*F5(1)
      Q3=Q(3)+5.0*(F5(1)+F5(2))
      Q4=Q(4)+5.0*(F5(1)+F5(2)+F5(3))
      PAR0=Q1/Q2
      PAR1=Q2/Q3
      PAR2=Q3/Q4
  400 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CTFERT(FS,AGE,RATE,DEG)
C
C     CALCULATE LN(-LN) VALUES AND DIFFERENCE
      COMMON NRUN,NUM,IPRINT
      DIMENSION FS(8),AGES(8)
      DIMENSION FF(8),Y(8),D(7)
      DIMENSION DD(5),YY(7)
C
      DATA AGES/≠10-14≠,≠15-19≠,≠20-24≠,≠25-29≠,≠30-34≠,≠35-39≠,≠40-44≠,
     2≠45-49≠/
C
C
C     CASE WHERE FIRST AGE GROUP HAS ZERO FERTILITY
      F1=FS(1)
      IF(F1.GT.0.0) GO TO 20
      FF(1)=0.0
      Y(1)=99.99999
      GO TO 4
   20 CONTINUE
C
C     CASE WHERE FIRST AGE GROUP HAS POSITIVE FERTILITY
      FF(1)=FS(1)
      Y(1)=ALOG(ALOG(FF(1))*(-1.0))
    4 CONTINUE
C
C     2ND TO 7TH AGE GROUPS
      DO 5 I=2,7
      FF(I)=FF(I-1)+FS(I)
      Y(I)=ALOG(ALOG(FF(I))*(-1.0))
    5 CONTINUE
C
C     CASE WHERE CUMULATED FERTILITY ≥ 1.0
      FF(8)=FF(7)+FS(8)
      F8=FF(8)
      IF(F8.LT.1.0) GO TO 21
      Y(8)=-99.99999
      GO TO 22
   21 CONTINUE
      Y(8)=ALOG(ALOG(FF(8))*(-1.0))
   22 CONTINUE
C
C                     CALCULATE DIFFERENCES IN Y VALUES.
      IF(F1.EQ.0.0) GO TO 32
      D(1)=Y(1)-Y(2)
      GO TO 30
   32 CONTINUE
      D(1)=99.99999
   30 DO 31 I=2,7
      II=I+1
      D(I)=Y(I)-Y(II)
   31 CONTINUE
C                     CALCULATE DIFFERENCES IN D VALUES
      DO 33 I=1,5
      II=I+1
      DD(I)=D(I)-D(II)
   33 CONTINUE
C                     ARRANGE VALUES OF Y IN ASCENDING ORDER OMITTING

C                     VALUE FOR 45-49
      DO 50 I=1,7
      J=8-I
      YY(J)=Y(I)
   50 CONTINUE
```

```
C                    WRITE ONTO TAPE1 THE DATA NEEDED BY MINUITS
      WRITE(1,200) AGE,RATE,DEG
 200  FORMAT(*AGE=*,F5.1,*R=*,F5.2,*M=*,F5.2)
      WRITE(1,204)
 204  FORMAT(9X,*1ALPHA*,5X,*0.0*,7X,*0.001*,5X,*-1.0*,6X,*1.0*)
      WRITE(1,205)
 205  FORMAT(9X,*2BETA*,6X,*1.0*,7X,*0.001*,5X,*0.75*,6X,*1.25*)
      WRITE(1,201)
 201  FORMAT(1X)
C               WRITE HEADING FOR USE OF FCN AND THEN
C                    WRITE ONTO TAPE1 THE YY-VALUES TO BE USED IN THE REGRESSION
C                    EQUATION, I.E. AGES 15-44
      WRITE (1,200) AGE, RATE, DEG
      WRITE(1,51) (YY(I),I=1,6)
  51  FORMAT(6F10.5)
      WRITE(1,202)
 202  FORMAT(*MINIMIZE*)
      IF (NUM.EQ.NRUN) GO TO 206
      CONTINUE
      WRITE(1,203)
 203  FORMAT(*END*)
      GO TO 207
 206  WRITE(1,208)
 208  FORMAT(*EXIT*)
 207  CONTINUE
C                    PRINT OUT THE RESULTS
      IF (IPRINT) 40 ,40,41
  41  CONTINUE
      PRINT 99
      DO 8 I=1,5
      PRINT 98,AGES(I),FS(I),FF(I),Y(I),D(I),DD(I)
   8  CONTINUE
  98  FORMAT(1H ,10X,A5,2(F12.5,F14.5),F14.5)
      PRINT 98,AGES(6),FS(6),FF(6),Y(6),D(6)
      DO 9 I=7,8
      PRINT 98,AGES(I),FS(I),FF(I),Y(I)
   9  CONTINUE
  99  FORMAT(1H0,41X,*GROUP*,7X,*CUMULATED*/32X,*AGE*,2(5X,*FERTILITY*),
     15X,*LN(-LN)*,5X,*DIFFERENCE*,5X,*DIFF(DIFF)*/
     2                                 43X,*FS*,12X,*FF*,12X,*Y*,13X,*D*,
     3               12X,*DD*)
  40  CONTINUE
      JAGE =AGE*10
      JRATE=RATE*10
      JDEG=DEG*10
      WRITE(7,300)     JAGE,JRATE,JDEG,(Y(I),I=1,7)
 300  FORMAT(I3,2I2,*2*,2X,7F9.5)
      RETURN
      END


      FUNCTION MITCH (X, RATE)
C
C
      CONS = 0.19465 / RATE
      U = -0.174 / RATE
      V = -0.2881 / RATE
      W = 6.06 * RATE
      MITCH = 0.0
      RETURN
C
      ENTRY FOP
      MITCH = CONS * EXP(U * (X-W) - EXP(V * (X-W)))
      RETURN
      END
```

```
AGE=10.0      MEAN=26.60     P0/P1= .0496      RHO=  .89     ***
  K=   .10       SD= 8.26    P1/P2= .3069     SMAM=11.14
  M=   .20    SKEW=   .279   P2/P3= .5693

                                    GROUP         CUMULATED
                           AGE    FERTILITY       FERTILITY       LN(-LN)       DIFFERENCE
                                      FS              FF             Y              D
                          10-14     .06746          .06746         .99188         .67931
                          15-19     .16743          .25489         .31256         .58226
                          20-24     .21109          .46598        -.26969         .59116
                          25-29     .18923          .65521        -.86086         .72076
                          30-34     .15891          .81412       -1.58162        1.09929
                          35-39     .11967          .93379       -2.68090        2.11520
                          40-44     .05798          .99177       -4.79610
                          45-49     .00823         1.00000      -99.99990
```

```
AGE=10.0      MEAN=25.92     P0/P1= .0496      RHO=  .90     ***
  K=   .10       SD= 8.05    P1/P2= .3071     SMAM=11.14
  M=   .40    SKEW=   .360   P2/P3= .5734

                                    GROUP         CUMULATED
                           AGE    FERTILITY       FERTILITY       LN(-LN)       DIFFERENCE
                                      FS              FF             Y              D
                          10-14     .07272          .07272         .96360         .70760
                          15-19     .20207          .27479         .25600         .62054
                          20-24     .22453          .49932        -.36454         .63097
                          25-29     .19174          .69106        -.99551         .75726
                          30-34     .14984          .84089       -1.75277        1.13676
                          35-39     .10502          .94592       -2.88953        2.15801
                          40-44     .04768          .99360       -5.04754
                          45-49     .00640         1.00000      -99.99990
```

```
AGE=10.0      MEAN=25.30     P0/P1= .0496      RHO=  .91     ***
  K=   .10       SD= 7.82    P1/P2= .3073     SMAM=11.14
  M=   .60    SKEW=   .433   P2/P3= .5775

                                    GROUP         CUMULATED
                           AGE    FERTILITY       FERTILITY       LN(-LN)       DIFFERENCE
                                      FS              FF             Y              D
                          10-14     .07784          .07784         .93731         .73540
                          15-19     .21629          .29413         .20191         .66000
                          20-24     .23714          .53127        -.45809         .67307
                          25-29     .19295          .72422       -1.13116         .79615
                          30-34     .14033          .86456       -1.92731        1.17631
                          35-39     .09155          .95611       -3.10362        2.20226
                          40-44     .03894          .99505       -5.30589
                          45-49     .00495         1.00000      -99.99990
```

APPENDIX 3.3

THE ORDINARY GOMPERTZ FIT TO STANDARD FERTILITY

In fitting the ordinary Gompertz curve to standard fertility, four parameters were allowed to vary to ensure as good a fit as possible. Hence the function is

$$F_s(x) = F_s \, C^{D^{x-x_o}}$$

where $F_s(x)$ is cumulative fertility, F_s is "completed fertility" (see later) and x_o is the origin of the age scale, x. C and D are parameters to be estimated, along with F_s and x_o.

The fitting procedure used is iterative, employing the computer program MINUIT (James and Roos, 1971). The objective function is the sum of squares,

$$\sum_x (f_s(x) - \hat{f}_s(x))^2$$

where $f_s(x)$ is standard age specific fertility and $\hat{f}_s(x)$ its estimate. Fitting was carried out in age specific rather than cumulative fertility so that an equally good fit could be obtained over the whole curve. (In cumulative fertility, where deviations cancel each other out, it is harder to detect differences from the standard at later ages.) Five year age groups were used rather than single years because this is the form used in the analyses.

The obtained fit is shown in Table A3.3.1 and is described by

$$\hat{F}_s(x) = 1.05374 \, . \, 0.04808^{0.8748^{x-16.732}}$$

The estimate of F_s($\hat{F}_s$ = 1.05374) is a scale parameter introduced in order to obtain a better fit than could be obtained by restricting this parameter to 1. In using the standard parameters, however, $\hat{F}_s$ is regarded as unity. The estimate of x_o = 16.732 is the value for which the best fit is obtained. The parameter, C, is dependent on this value of x_o.

These estimates of the ordinary Gompertz parameters of the standard fertility distribution are used to obtain initial estimates of the parameters of the transformed Gompertz model, as described in Chapter 5. Though based on endpoint data, they are used in the analysis of both endpoint and midpoint data. Any change in these parameter estimates that might occur if they were based on midpoint values of the standard would be small (since the two sets of parameters estimate the same curve) and not of importance.

Table A3.3.1: The Gompertz fit to age specific standard fertility

Age	Standard	Fit	Difference
10-14	.00277	.02247	-.02020
15-19	.13307	.12543	.00764
20-24	.24147	.23761	.00386
25-29	.23130	.24392	-.01262
30-34	.18757	.17965	.00792
35-39	.13401	.11105	.02296
40-44	.06169	.06267	-.00098
45-49	.00812	.03374	-.02562

Table A3.3.1: The Gompertz fit to age specific standard fertility

Age	Standard	Fit	Difference
10-14	.00277	.02247	-.02020
15-19	.13307	.12543	.00764
20-24	.24147	.23761	.00386
25-29	.23130	.24392	-.01262
30-34	.18757	.17965	.00792
35-39	.13401	.11105	.02296
40-44	.06169	.06267	-.00098
45-49	.00812	.03374	-.02562

APPENDIX 4.1

DETERMINATION OF TEENAGE FECUNDABILITY FOR USE IN THE MODIFIED BARRETT SIMULATION MODEL

The determination of a fecundability function at early ages was based on empirical evidence about teenage probabilities of conception and on the shape of age specific fertility distributions of the kind of populations of interest. In particular, fecundability was chosen to produce fertility schedules typical of high fertility populations. Comparison was therefore made with the fertility pattern of the standard developed in Chapter 3 and with the empirical average used in Appendix A4.2. In all cases comparison is between the patterns of fertility only and to this intent all schedules are normalised to sum to unity.

The first function to be tried was $\rho = .0001$ at age 10, monotonically increasing to the predetermined basic fecundability, ρ^*, (for a noncontracepting population) at age 20. (The value of .0001 at age 10, rather than zero, is for convenience in computing.) This function was based on empirical evidence from various studies considering time to conception (Gray, 1977) suggesting that at age 15, the interval to first conception is about twice as long as at age 20, and that fecundability at 15 is roughly half that at 20 years. Ratios of fertility rates for ages 10-14 and 15-19 to the rate for 20-24 were calculated and are shown in Table A4.1.1 for several sets of nuptiality parameters. These ratios and those for values of ρ at age 10 of .05 and .025, indicated that the required proportions of fertility at ages 10-14 and 15-19 could not be adequately achieved by a linear fecundability function, and that an exponential function might be more appropriate. This is not surprising since the empirical evidence is for married women. As mentioned in Chapter 4, the required teenage fecundability function should account for both teenage subfertility and age at menarche. An exponential function would be appropriate because of the extent to which menarche precedes marriage.

The first exponential function to be tried, obtained from rough estimates of fertility at young ages, was

$$\rho = \rho^* \exp\{(x - 260)/65\}$$

where x is age in lunar months. The schedules in Table A4.1.2 show that the resulting fertility was too low. Replacement of the constant value, 65(= b), by smaller values (50 and 40) resulted in the final choice of

$$\rho = \rho^* \exp\{(x - 260)/40\}$$

as the teenage fecundability function.

Table A4.1.1: Fertility patterns resulting from linear teenage fecundability functions

Age	ρ = .0001				ρ = .05		ρ = .025		Standard	Empirical average
10-14	.02320	.01188	.00858	.00499	.01828	.01290	.01530	.00942	.00277	-
15-19	.14763	.11658	.12130	.12712	.12318	.12644	.11902	.12927	.13307	.09926
20-24	.19041	.19403	.19446	.19715	.19092	.19404	.19109	.19623	.24147	.25628
25+	.63876	.67751	.67566	.67074	.66762	.66662	.67459	.66508	.62269	.64446
					Ratios of rates to 20-24 rate					
10-14	.12	.06	.04	.03	.10	.07	.08	.05	.01	-
15-19	.78	.60	.62	.64	.65	.65	.62	.66	.55	.39
20-24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
					Nuptiality parameters					
a_0	10.0	10.0	11.0	12.0	10.0	11.0	10.0	11.0	-	-
k	0.4	0.6	0.5	0.4	0.6	0.5	0.6	0.5	-	-

Table A4.1.2: Fertility patterns resulting from exponential teenage fecundability functions

Age	b=65		b=50	b=40
10-14	.01511	.00977	.00868	.00689
15-19	.13805	.14444	.11978	.11007
20-24	.24894	.24814	.22691	.23393
25+	.59790	.59765	.64463	.64911
	Ratios of rates to 20-24 rate			
10-14	.06	.04	.04	.03
15-19	.55	.58	.53	.47
20-24	1.00	1.00	1.00	1.00
	Nuptiality parameters			
a_0	10.0	11.0	10.0	10.0
k	0.6	0.5	0.6	0.6

APPENDIX 4.2

MODIFICATION OF THE STERILITY FUNCTION

The original sterility function used in the Barrett simulation model was based on data from the 1911 Census of Ireland in the form of marriage cohorts surviving to the end of childbearing. The function is

$$x_s = 28 + z/0.012 \qquad \text{A4.2.1}$$

where x_s is age at sterility and z is a random variable between 0 and 1. A constant proportion of women, 4.8%, is assumed sterile at ages less than 28 years.

In using the simulation model to reproduce fertility schedules for all women, some means was necessary of allowing for marital dissolution. The simplest method of doing this was to incorporate marital dissolution into the sterility function, thereby producing a combined sterility/marital dissolution function (hereafter referred to as sterility). The development of this combined function is described below. Throughout, the same constant proportion, 4.8%, is assumed sterile up to the age at which the function applies.

In testing the various sterility functions, the effects of other parameters were avoided where possible. Thus all schedules used in the comparisons below are for

a_o = 10.0 years and k = 0.6. This combination of a_o and k means that 83% of marriage is completed by age 20, and 93% by age 25, thereby largely avoiding the effect of marriage on fertility rates at ages of interest. The schedules are calculated with a post partum coefficient, r, of 1/6, appropriate for the noncontracepting situation. The teenage fecundability function changes because its own development took place concurrently with the development of the sterility function. Teenage fecundability, however, has no bearing on the pattern of sterility or on the pattern of fertility at ages over 20.

The first modification to be made to equation A4.2.1 was to increase the rate at which sterility occurs by increasing the constant 0.012 (= s_1). Table A4.2.1 gives the resulting schedules for several values of s_1. Ratios of rates at ages above 25 to the rate for 20-24 are also shown. Comparison of these ratios with standard fertility (developed in Chapter 3) and with empirical evidence (the average of 18 schedules from censuses and surveys given in Table A4.2.2) indicated that the reduction in fertility due to increased s_1 values is too great at ages 35+ compared to that at ages 25-34. This suggests that a linear function is not appropriate for modified sterility purposes, and that an exponential function might produce a better fit.

The first exponential function to be tried, based on rough calculations, was

Table A4.2.1: Fertility schedules resulting from different values of s_1 in the sterility function $x_s = 28 + z/s_1$

	Value of s_1						
Age	.020	.030	.040	.050	.060	Standard fertility	Empirical average
10-14	.01242	.01527	.01494	.01614	.01499	.00277	-
15-19	.12520	.13596	.14105	.15229	.15553	.13307	.09926
20-24	.19546	.21639	.22681	.24843	.25705	.24147	.25628
25-29	.21217	.23004	.24247	.25137	.26434	.23130	.25022
30-34	.18421	.18098	.18959	.19123	.19171	.18757	.19238
35-39	.14041	.12740	.11323	.09895	.09275	.13401	.12965
40-44	.09146	.06873	.05697	.03747	.02349	.06169	.05426
45-49	.03449	.02314	.01421	.00413	.00014	.00812	.02149
49+	.00418	.00208	.00072	-	-	-	-
	Ratios of rates to 20-24 rate						
20-24	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25-29	1.09	1.06	1.07	1.01	1.03	.96	.98
30-34	.94	.84	.84	.77	.75	.78	.75
35-39	.72	.59	.50	.40	.36	.55	.51
40-44	.47	.32	.25	.15	.09	.26	.21
45-49	.18	.11	.06	.02	.00	.03	.08
49+	.02	.01	.00	-	-		

Table A4.2.2: Age specific fertility schedules used to compute an empirical average pattern of fertility

Source	Country	Year	15-19	20-24	25-29	30-34	35-39	40-44	45+
1	Mauritius	1966	07729	26000	25650	20347	14204	05368	00701
1	Chile	1952	08285	22160	24522	21068	14599	07371	01994
1	Japan	1950	01882	22077	32450	24000	14343	04932	00316
1	Malaysia	1966	06859	23263	26318	22127	13853	06012	00157
1	Iceland	1963	10490	29673	25829	18317	10942	04585	00163
1	Liberia	1970	17305	20455	18900	19545	12903	08461	02432
1	Tunisia	1970	03802	20856	24813	22465	16890	07907	03268
1	Bahamas	1970	11164	28859	26455	16203	11903	04778	00637
1	Guatemala	1970	11645	24183	22624	18794	14903	06083	01768
1	Panama	1966	13376	28302	25320	16949	11568	03796	00707
1	Turkey	1967	07642	25299	24517	19683	14086	05653	03119
2	Bangladesh (total)	1974	11052	23369	21971	18575	13648	06591	04794
2	Bangladesh (rural)	1974	11025	23509	21877	18468	13599	06653	04869
2	Bangladesh (urban)	1974	11957	22271	23342	21208	14134	04318	02770
3	Barbados	1969	16345	26534	22837	17395	12104	04372	00413
3	Fiji	1971	07144	31155	28284	16947	11165	03598	01707
4	Fiji (Fijians)	1974	06951	24103	26794	22646	13789	05717	-
4	Fiji (Indians)	1974	10734	35311	28672	14407	07486	03390	-
	Average		09926	25628	25022	19238	12965	05426	02149

Source: 1. United Nations. Demographic Yearbook, various years. New York.
2. Report on the 1974 Bangladesh Retrospective Survey of Fertility and Mortality, 1977. Population Bureau, Ministry of Overseas Development, London; and Census Commission, Statistics Division, Ministry of Planning, Dacca.
3. Personal collection of Dr J.G.C. Blacker.
4. The Fiji Fertility Survey 1974: A Summary of Findings. World Fertility Survey, 1977.

Note: Bangladesh and Fiji are over-represented to give weight to the high late fertility in Bangladesh, and to the different fertility patterns in Fiji.

$$x_s = 28 + 10 \ln (1 + 9z)$$

allowing sterility to occur between 28 and 52 years. The constant, 10 (= c) was later reduced in an attempt to reduce fertility at older ages in relation to the middle childbearing ages. It was found necessary, however, to reduce age at which sterility begins from 28 (= a). Various combinations of a and c were tried resulting in the schedules shown in Table A4.2.3. The final combination was a = 20, c = 13 such that

$$x_s = 20 + 13 \ln (1 + 9z)$$

is the final modified sterility function.

$$x_s = 28 + 10 \ln (1 + 9z)$$

allowing sterility to occur between 28 and 52 years. The constant, 10 (= c) was later reduced in an attempt to reduce fertility at older ages in relation to the middle childbearing ages. It was found necessary, however, to reduce age at which sterility begins from 28 (= a). Various combinations of a and c were tried resulting in the schedules shown in Table A4.2.3. The final combination was a = 20, c = 13 such that

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is the final modified sterility function.

Table A4.2.3: Fertility schedules resulting from different combinations of a and c in the sterility function $x_s = a + c \ln (1 + 9z)$

Age	a = 28 c = 10	a = 28 c = 9	a = 28 c = 8	a = 28 c = 7	a = 26 c = 8	a = 23 c = 8	a = 23 c = 11	a = 22 c = 12	a = 21 c = 12	a = 20 c = 13
10-14	.01365	.01132	.01335	.01214	.00649	.00577	.00675	.00743	.00672	.00533
15-19	.12443	.11492	.12677	.13370	.11884	.12864	.10839	.10699	.11315	.11034
20-24	.20500	.21806	.22105	.23893	.24947	.28035	.24150	.23348	.23758	.24327
25-29	.21507	.22777	.23889	.24665	.25662	.28109	.24290	.23386	.23847	.23807
30-34	.19991	.20293	.20782	.21009	.21198	.20893	.19870	.20038	.20603	.19464
35-39	.14252	.14391	.13445	.12585	.12043	.08798	.13361	.13845	.12950	.13205
40-44	.07548	.06976	.05293	.03263	.03590	.00725	.06050	.06709	.05993	.06145
45-49	.02285	.01132	.00473	-	.00026	-	.00764	.01221	.00862	.01460
50+	.00108	-	-	-	-	-	-	.00013	-	.00025
Ratios of rates to 20-24 rate										
20-24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
25-29	1.05	1.04	1.08	1.03	1.03	1.00	1.01	1.00	1.00	.98
30-34	.98	.93	.94	.88	.85	.75	.82	.85	.87	.80
35-39	.70	.66	.61	.53	.48	.31	.55	.59	.55	.54
40-44	.37	.32	.24	.14	.14	.03	.25	.29	.25	.25
45-49	.11	.05	.02	-	.00	-	.00	.05	.04	.06
50+	.01	-	-	-	-	-	-	.00	-	.00

APPENDIX 4.3

THE EFFECT ON FERTILITY OF VARIATION AMONG WOMEN WITH RESPECT TO DESIRED FAMILY SIZE

The use of a fixed value of the desired family size parameter, DFS, in the simulation of fertility schedules is questionable in light of the results of studies giving distributions of ideal or desired family sizes (for example Knodel and Prachuabmoh, 1973; Freedman, 1963). In order to determine whether to introduce variability among women with respect to DFS, several simulations were run with combinations of DFS for comparison with a fixed value.

The chosen combinations were 4, 5, 6 and 4, 6 to be compared with 5. In order to make the results of the combinations directly comparable with DFS = 5 results, weights were used to ensure the same completed fertility. Hence, for low contraceptive effectiveness ($E_1 = 0.7$, $E_2 = 0.9$ with $r = 1/6$) and for nuptiality parameters $a_o = 130$ months, $k = 0.6$, the weights were calculated to produce a completed fertility rate of 5.19 (see Table 4.7). Since the completed fertility rates for DFS = 4 and DFS = 6 (under the same parameter conditions) are 4.68 and 5.84 respectively, weights for the 4,6 combination were chosen to satisfy:

$$\begin{aligned} 4.68\, w_4 + 5.84\, w_6 &= 5.19 \\ w_4 + w_6 &= 1 \end{aligned}$$

giving $w_4 = .56$ and $w_6 = .44$. In the simulation, therefore, 56% of women had a DFS of 4, and 44% had a DFS of 6. For the 4, 5, 6 combination, 50% of women were assumed to have a desired family size of 5. The weights were therefore calculated to satisfy:

$$\begin{aligned} 4.68\, w_4 + \tfrac{1}{2}\, 5.19 + 5.84\, w_6 &= 5.19 \\ w_4 + \tfrac{1}{2} \qquad + \qquad w_6 &= 1 \end{aligned}$$

The solution to these equations is $w_4 = .28$ and $w_6 = .22$; the distribution of DFS thus being 28%, 50% and 22% for 4, 5 and 6 respectively.

The results of these runs are shown in the first three columns of Table A4.3.1. The slight differences in completed fertility are due to sampling error. Changes in the age pattern of fertility are small as can be seen in the lower half of the table (the larger changes in the tails of the distribution are a result of the small numbers on which the rates are based).

The right hand half of Table A4.3.1 gives results for high contraceptive effectiveness ($E_1 = 0.9$, $E_2 = 0.99$, $r = .3$, with $a_o = 130$ months, $k = 0.6$ as before) where the effect of a variable DFS parameter might be expected to be greater because of the greater control over achieved fertility. The results show that this is not the case, however. Weights for these runs were calculated using completed fertilities of 3.78, 4.63 and 5.32 for

Table A4.3.1: The effect of a variable desired family size parameter on the pattern of fertility

	Low contraception			High contraception		
Age	4,5,6	4,6	5	4,5,6	4,6	5
10-14	.00808	.01259	.00984	.01109	.00942	.01012
15-19	.16631	.17830	.16721	.20209	.20267	.20301
20-24	.34264	.32513	.34311	.41658	.40578	.40840
25-29	.24389	.24828	.25092	.23646	.23926	.24629
30-34	.14610	.13997	.13886	.09724	.09904	.09451
35-39	.06506	.06217	.06075	.02545	.03199	.02368
40-44	.02329	.02765	.02430	.00827	.00942	.01119
45-49	.00462	.00591	.00501	.00261	.00219	.00258
50-54	-	-	-	.00022	.00022	.00022
level	5.20	5.24	5.19	4.60	4.65	4.63
	Ratios of combination rates to DFS = 5 rate					
10-14	.82	1.30	1.00	1.10	.93	1.00
15-19	.99	1.07	1.00	1.00	1.00	1.00
20-24	1.00	.95	1.00	1.02	.99	1.00
25-29	.97	.99	1.00	.96	.97	1.00
30-34	1.05	1.01	1.00	1.03	1.05	1.00
35-39	1.07	1.02	1.00	1.07	1.35	1.00
40-44	.96	1.14	1.00	.74	.84	1.00
45-49	.92	1.18	1.00	1.01	.85	1.00
50-54	-	-	-	1.00	1.00	1.00

DFS = 4, 5 and 6 respectively. For the 4, 6 combination the weights satisfy:

$$\begin{aligned} 3.78\, w_4 + 5.32\, w_6 &= 4.63 \\ w_4 + w_6 &= 1 \end{aligned}$$

giving $w_4 = .45$ and $w_6 = .55$. For the 4, 5, 6 combination, the weights were $w_4 = .22$, $w_5 = .50$ and $w_6 = .28$.

The data in Table A4.3.1 indicate that at this level of fertility, changes in the pattern of fertility due to the introduction of variability among women in the desired family size parameter are not large enough to be of significance, especially in view of sampling errors. It was therefore decided to leave DFS as a fixed parameter among women. It must be noted, however, that for smaller family sizes with the prerequisite very high levels of contraceptive effectiveness, the chance element would be much less important and variability in DFS would make a significant difference to fertility patterns.

APPENDIX 4.4

READOPTION OF THE BARRETT STERILITY FUNCTION TO PRODUCE MARITAL FERTILITY RATES

The modified Barrett simulation model used to generate declining fertility produces age specific fertility rates for ever-married women. To be able to compare these rates with Knodel's work (see Chapter 4), a means is needed of generating marital fertility corresponding to the already simulated fertility schedules comprising the fertility decline.

Barrett's original simulation model has no provision for marital dissolution, and therefore calculates rates for currently married women where all women survive to the end of the childbearing period. His sterility function, describing age, x_s, at biological sterility is

$$x_s = 28 + z/0.012$$

where z is a random variable between 0 and 1. In the modified model, marital dissolution was incorporated into the "sterility" function such that

$$x_s = 20 + 13 \ln(1 + 9z)$$

where x_s now represents age at sterility or marital

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dissolution, whichever occurs first. Removal of this function, and its replacement by Barrett's original sterility function would therefore produce the required marital fertility schedules (also taking proportions ever-married into account). These are reproduced in Table A4.4.1 for the 5 stages of the simulated fertility decline. Values of m, the index of voluntary fertility control in the Coale-Trussell model, are also presented. Comparison of these values for Stage 1 of the decline with Knodel's values of m, reproduced from Knodel (1977) in Table A4.4.2, shows that the degree of fertility control at the beginning of the fertility decline is rather high and that Stage 1 is, in fact, already into a transition in fertility. This is not surprising since the desired family size for this schedule is 6, and it is recognised that there is quite a large gap in fertility level and pattern between this schedule and the one resulting from an unattainable desired family size. (The values of m for this latter schedule, however, are -0.19, -0.05, 0.02, -0.17, -0.77 for age groups 25-29 to 45-49 respectively, with $\bar{m} = -0.23$ and $\sigma_m = 0.31$. Such negative m values indicate very high fertility patterns of such populations as the Hutterites, whose marital fertility is higher than standard natural fertility. These populations are not of particular interest here.)

The standard deviations, σ_m, in Table A4.4.1 are also considerably higher than those calculated by Knodel. The reason for these high values lies in the

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Table A4.4.1: Age specific marital fertility at each stage of the fertility decline using the Barrett sterility function

	Stage of fertility decline				
Age	1	2	3	4	5
10-14	.03715	.03468	.02987	.01206	.02864
15-19	.19539	.19352	.21549	.19188	.21539
20-24	.27946	.28034	.32053	.34214	.34308
25-29	.22795	.23632	.22016	.23396	.22661
30-34	.14228	.13729	.11837	.12511	.11836
35-39	.07304	.07257	.06055	.05933	.04379
40-44	.03278	.03139	.02617	.02687	.01808
45-49	.00980	.01250	.00746	.00678	.00526
50-54	.00215	.00140	.00140	.00187	.00079
level	7.45	7.84	7.14	6.96	6.33
%	100	105	96	93	85
	m values				
25-29	.50	.38	1.11	1.13	1.25
30-34	.77	.83	1.25	1.26	1.35
35-39	.95	.96	1.26	1.34	1.63
40-44	.80	.83	1.06	1.08	1.37
45-49	.23	.09	.48	.57	.73
$\bar{m}$	.65	.62	1.03	1.08	1.27
σ_m	.29	.37	.32	.30	.33
$\bar{m}'$	.75	.75	1.17	1.20	1.40
σ_m'	.19	.25	.10	.12	.16

Table A4.4.2: Index of fertility control, m, and associated statistics for selected European and Asian populations. (Reproduced from Knodel, 1977)

	Date	Source	M (Index of fertility level)	m (Index of fertility control)		m for individual age groups				
				Mean	Standard deviation	25–29	30–34	35–39	40–44	45–49
European populations										
Reconstitution studies										
7 south and central French villages	17–18th Century	A	0·94	0·02	0·04	−0·11	0·05	0·07	0·07	n.a.
6 north French villages	17–18th Century	A	1·15	0·01	0·08	−0·09	−0·05	0·01	0·12	n.a.
14 NW French villages	17–18th Century	A	1·06	0·03	0·04	−0·03	0·04	0·06	0·06	n.a.
8 German villages	17–19th Century	A	0·97	−0·00	0·04	−0·06	0·01	0·02	0·03	n.a.
Quebec	17th Century	B	1·11	−0·06	0·08	−0·00	−0·13	−0·14	−0·11	0·06
Alskog (Swedish village)	1745–1820	S	0·79	0·13	0·10	−0·04	0·22	0·19	0·20	0·02
National statistics										
Bulgaria	1901–05	C	0·83	0·02	0·34	0·36	0·05	0·24	0·09	−0·63
Denmark	ca 1865*	D	0·97	0·26	0·05	0·26	0·33	0·30	0·19	0·23
Finland	1871–80	E	0·98	0·24	0·13	0·43	0·28	0·19	0·07	n.a.
Norway	1871–75†	F	0·93	−0·05	0·21	0·06	0·14	0·06	−0·05	−0·46
Sweden	1751–1800	G	1·00	0·23	0·20	0·45	0·28	0·33	0·22	0·14
Asian populations										
China – rural	1930	H	0·57	0·06	0·09	−0·05	0·08	0·15	0·13	−0·04
Comilla (Bangladesh)	1963–64	I	0·71	0·13	0·06	0·09	0·06	0·22	0·15	n.a.
Hong Kong	1961	J	1·02	0·61	0·11	0·55	0·68	0·32	0·66	0·42
Mysore (India)‡		K	0·68	0·24	0·22	−0·10	0·42	0·21	0·44	n.a.
Indonesia – rural§	1965–70	L	0·77	0·17	0·06	0·17	0·23	0·25	0·13	0·08
4 Japanese villages	17–19th Century	M	0·84	0·18	0·19	0·44	0·27	0·23	0·12	−0·14
Japan	1925	N	0·74	0·21	0·03	0·25	0·21	0·19	0·24	0·18
Korea	1961	O	0·81	0·03	0·08	−0·10	0·09	0·12	0·07	−0·05
Malaysia	1957	P	0·86	0·25	0·25	0·31	0·41	0·40	0·34	−0·20
Pakistan	1963–65	Q	0·64	−0·24	0·41	−0·07	−0·06	0·03	−0·04	−1·06
Philippines	1963–67	R	0·94	0·19	0·08	0·17	0·25	0·24	0·26	0·04
Singapore	1957	P	1·00	0·30	0·08	0·26	0·32	0·36	0·35	0·19
Sri Lanka	1953	P	0·86	0·44	0·28	0·20	0·28	0·45	0·78	0·47
Taiwan	1956	P	0·82	−0·02	0·13	−0·08	0·05	0·09	0·07	−0·25
Thailand	1960	P	1·02	0·11	0·23	0·31	0·29	0·15	0·12	−0·34

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Notes: n.a. = Not available.

* Estimated from rates computed for birth cohorts. † rate for women aged 45–49 calculated by author. ‡ excluding Bangalore City. § based on unweighted mean of separate marital fertility schedules for rural areas of major regions.

45-49 age group m values, which are consistently much lower than the values at other ages. Recalculation of both $\bar{m}$ and σ_m without this last age group ($\bar{m}'$ and σ_m' in Table A4.4.1) reduces the standard deviations to a level which is well within Knodel's range of findings, (but obviously increases $\bar{m}$). The low values of m at age 45-49 indicate higher fertility at these late ages than would be expected from the pattern of fertility at ages less than 45. This phenomenon is also found in the schedule resulting from an unattainable desired family size and cannot therefore be attributed to the way in which fertility has been made to decline in the simulation exercise. (The m values for this schedule are given above; removal of the final value of -0.77 increases the mean to $\bar{m}' = -0.10$, and decreases the standard deviation to $\sigma_m' = 0.10$.) This high late fertility can partly be explained by the intentional overrepresentation of high fertility at late ages in both the empirical average (see Appendix 4.2) and in the standard fertility schedule (developed in Chapter 3), both of which were used as references in determining a suitable sterility function for the simulation of age specific fertility rates for all (or ever-married) women. The phenomenon can also be partly attributed to the sterility and menopause functions incorporated into the simulation model, which are based on the fertility experience of women who had married at ages 20-24 and had reached the end of the child-bearing period by the 1911 Census of Ireland. Since the pattern of fertility for all women is consistent with empirical evidence (as shown in Appendix 4.2) which is not biased sufficiently towards late high fertility to account for the size of the fall in m values at late ages, the sterility function used to produce these marital fertility schedules must be inappropriate for present purposes (though it is appropriate for the Irish data). This is confirmed by Barrett who acknowledges (verbally) that sterility at late ages is underestimated by the function.

Further modification of the sterility function to correct its underestimation at late ages would be desirable if the simulation of marital fertility were of particular interest. It is sufficient here, however, to show that the values of m are reasonably consistent within schedules, that is that their standard deviations are small. The age patterns of fertility are thus shown to be satisfactory.

APPENDIX 4.5

THE VALIDITY OF THE AGE PATTERN OF DECLINE IN THE SIMULATED FERTILITY DECLINE

Knodel (1977) has documented the age pattern of fertility decline for Asian and pre-industrial European populations. The patterns are described by calculation of a series of percentage changes in marital fertility for each age group. Values for Knodel's Asian populations are reproduced in Table A4.5.1, and corresponding values for the simulated decline are shown in Table A4.5.2. These latter values are calculated from the marital age specific fertility rates obtained by using the Barrett sterility function in the simulation program. These rates have been shown to be rather high at very late ages (see Appendix 4.4): however, this should have a negligible effect on the percentage changes in rates. (In fact, if it has any effect at all, it serves to reduce the percentage changes slightly.)

Knodel notes a general increase in the percentage decline in fertility with age, though Malaysia, Sri Lanka and Thailand are noted as being rather erratic. Leaving aside values for very early and very late ages where sampling errors are large, the simulated data produce the same age pattern for the two periods between stages 2 and 3 and stages 4 and 5. For the other periods, fertility has actually increased at most ages, though

Table A4.5.1: Percentage change in marital fertility at different ages for selected Asian countries. (Reproduced from Knodel, 1977)

	Hong Kong*				Japan			Korea			West Malaysia†		Singapore‡		
	1961 -66	1966 -71	1971 -74	1961 -74	1925 -40	1940 -70	1925 -70	1957/61 -62/66	1961 -68	1962/63 -70/71	1957 -67	1960 -70	1957 -66	1966 -70	1957 -70
								Source A	Source B	Source C	Source A	Source B			
15-19	−14	−7	n.a.	n.a.	−8	−18	−24	−18	−49	−3	+14	−6	+51	−9	+37
20-24	+7	−10	+15	+11	−3	+15	+2	+18	−2	+12	−11	+10	+13	−23	−14
25-29	−9	−10	−1	−18	−1	−12	−13	−11	−7	+5	−10	−14	−13	−21	−31
30-34	−13	−21	−15	−41	−6	−60	−62	−14	−30	−31	−15	−5	−27	−32	−50
35-39	−23	−26	−16	−52	−15	−87	−89	−31	−43	−46	−9	−13	−35	−42	−62
40-44	−31	−33	−26	−66	−17	−96	−96	−31	−37	−53	−21	−32	−38	−51	−70
45-49	−67	0	−30	−77	−26	−98	−98	−20	−57	−62	−35	−4	−42	−40	−65

	Sri Lanka			Taiwan					Thailand	
	1953 -63	1963 -71	1953 -71	1956 -59	1959 -64	1964 -69	1969 -74	1956 -74	1960- 69/70	1969/70 -71/72
15-19	+22	+18	+44	−15	−5	+40	+16	+32	−3	−1
20-24	0	−2	−2	+8	+7	+9	−6	+18	−6	−11
25-29	−2	−9	−10	+3	+1	−10	−18	−23	−10	0
30-34	−4	−12	−15	−5	−22	−30	−36	−67	−20	−23
35-39	+1	−10	−10	−12	−38	−48	−46	−83	−19	−16
40-44	+12	−8	+2	−19	−40	−57	−58	−91	+24	−19
45-49	−13	0	−13	−39	−44	−50	−60	−93	−15	−50

Sources:

Hong Kong: United Nations, Economic and Social Commission for Asia and the Pacific, *The Demographic Situation in Hong Kong*, ESCAP Country Monograph Series No. 1 (1974) and Mok, 1975.

Japan: Calculated from Table 8 in Kazumasa Kobayashi and Yoshihiro Tsubouchi, *Trends and Regional Variations of Marital Fertility in Japan*, Kyoto Conference on Fertility Transition (Honolulu: East-West Population Institute and Kyoto: Center for Southeast Asian Studies, 1975).

Korea: Source A – Lee-Jay Cho and Man Jun Hahm, 'Recent Change in Fertility Rates of the Korean Population', *Demography* 5 (1968) pp. 690–698; Source B – Robert J. Lapham and W. Parker Mauldin, 'National Family Planning Programs: Review and Evaluation', *Studies in Family Planning* 3, No. 3 (1972) pp. 29–52; Source C – KIFP Survey cited in Lee-Jay Cho, 'Current Fertility Estimates and Trends', *Population and Family Planning in the Republic of Korea*. (Seoul: Korean Institute for Family Planning, 1974), 2, Selection 36, pp. 401–410.

West Malaysia: Source A – United Nations, *Population Bulletin No. 7*. New York: (Department of Economic and Social Affairs, 1965); and Lee-Jay Cho, James A. Palmore and Lyle Saunders, 'Recent Fertility Trends in West Malaysia', *Demography* 5 (1968), pp. 732–744; Source B – Lee-Jay Cho and Robert Retherford, 'Comparative Analysis of Recent Fertility Trends in East Asia', in *International Population Conference, Liege, 1973*, vol. 2 (Liege: International Union for the Scientific Study of Population (1973)), pp. 163–178.

Singapore: United Nations, *Population Bulletin No. 7*. New York: Department of Economic and Social Affairs, 1965) and adapted from Saw Swee-Hock, *Population Policies and Fertility Decline in Singapore*. Kyoto Conference on Fertility Transition (Honolulu: East-West Population Institute, and Kyoto: The Center for Southeast Asian Studies, 1975).

Sri Lanka: United Nations, *Population Bulletin No. 7*. New York: Department of Economic and Social Affairs, 1965); and Dallas F. S. Fernando, 'Fertility Trends in Sri Lanka and Future Prospects', *Journal of Biosocial Science* 8 (1976), pp. 35–43.

Taiwan: United Nations, *Population Bulletin No. 7*. New York: Department of Economic and Social Affairs, 1965); and Taiwan, Ministry of the Interior, *passim*, *Taiwan Demographic Factbook*.

Thailand: United Nations, *Population Bulletin No. 7*. New York: Department of Economic and Social Affairs, 1965; and John Knodel and Pichit Pitaktepsombati, 'Fertility and Family Planning in Thailand: Results from Two Rounds of a National Study', *Studies in Family Planning* 6 (1975), pp. 402–413.

Notes:

* Rates for 1972–74 estimated by author by dividing the age-specific fertility rates by the proportion of women currently married in each age group.

† 1957 rates refer to all of Malaysia.

‡ 1966 and 1970 rates estimated by author by dividing age-specific fertility rates by the proportion currently married.

Table A4.5.2: Percentage change in marital fertility between stages of the simulated fertility decline

	Periods of fertility decline for stages:						
Age	1 to 2	2 to 3	3 to 4	4 to 5	1 to 3	3 to 5	1 to 5
10-14	-02	-22	-61	+116	-23	-15	-34
15-19	+04	+01	-13	+02	-06	-11	-06
20-24	+06	+04	+04	-09	+10	-05	+04
25-29	+09	-15	+04	-12	-08	-09	-16
30-34	+02	-22	+03	-14	-20	-11	-29
35-39	+05	-24	-04	-33	-21	-36	-49
40-44	+01	-24	+00	-39	-24	-39	-53
45-49	+34	-46	-11	-30	-27	-37	-54
50-54	-31	-09	+30	-62	-38	-50	-69

this is not surprising for the period between stages 1 and 2 since marital completed fertility increased anyway, and for the remaining period, between stages 3 and 4, marital completed fertility did not fall appreciably (see Table A4.4.1). The problem lies in the level of fertility rather than in the age pattern of change.

It is interesting to note that the large decreases in marital completed fertility occur where the desired family size parameter is decreased by one child, but that the level either increases or decreases only slightly when DFS remains unchanged over two consecutive stages. Since the effect of the parameter, k, affecting the pace of first marriage, is removed by virtue of the fact that the rates are for marital fertility, DFS is the most important parameter in determining the decline in fertility. The fact that it has not changed during two of the periods of decline has meant that marital fertility has not declined appreciably over these periods. (The increase in marital completed fertility at stage 2 is due to the effect of the other parameters, which may, in combination, increase fertility (see Chapter 4), and may also be caused to some extent by sampling errors.) The decline in completed fertility for all women was achieved at these stages by delayed marriage. Calculation of percentage changes between stages 1 and 3, stages 3 and 5 and stages 1 and 5 results in a clear pattern of decline of the type found by Knodel. These percentage changes are also shown in Table A4.5.2.

To consider the age pattern of fertility decline of the simulated rates for all women, parallel percentage declines were calculated for the rates appearing in Table 4.13. These rates, which incorporate the effect of delayed marriage on fertility, embody an uninterrupted decline in completed fertility, though the greatest decreases again appear where DFS is reduced. The percentage changes in age specific fertility appear in Table A4.5.3. The clear pattern present in Table A4.5.2 is obscured by the inclusion of all women (rather than currently married only), but negative values are introduced into the first and third columns.

These findings support the earlier result (Chapter 4) that DFS is by far the most influential variable in reducing simulated fertility. They indicate perhaps that some decrease in DFS should have been incorporated into each stage of the decline, and as such suggest that the periods between stages 1 and 3 and stages 3 and 5 might be of greater interest than those between stages 1 and 2, and stages 3 and 4.

Table A4.5.3: Percentage change in age specific fertility between stages of the simulated fertility decline

	Periods of fertility decline for stages:						
Age	1 to 2	2 to 3	3 to 4	4 to 5	1 to 3	3 to 5	1 to 5
10-14	-12	-72	+10	-64	-76	-60	-90
15-19	-21	-25	-24	-35	-41	-51	-71
20-24	-01	-09	-14	-27	-10	-37	-43
25-29	+03	-14	-05	-20	-12	-24	-33
30-34	-01	-22	+08	-23	-22	-17	-35
35-39	-02	-27	+00	-15	-28	-15	-39
40-44	-18	-13	-01	-28	-29	-28	-49
45-49	+28	-55	+58	-21	-43	+25	-29
50-54	-50	+100	-51	+02	-00	-50	-50

APPENDIX 4.6

THE COMPARISON OF LESTHAEGHE'S NUPTIALITY SCHEDULES WITH THOSE USED IN THE SIMULATED FERTILITY DECLINE

Using recent data for Maghreb and Middle East populations, Lesthaeghe (1971) obtained a schedule of proportions ever married to be used as the pretransitional schedule of a transitional nuptiality series. A post-transitional schedule was developed to be representative of moderately early marrying contemporary European populations. For both of these schedules, values of Coale's nuptiality parameters (a_0, k and C) were estimated (by the method described in Appendix 3.1A). A series of transitional nuptiality schedules was then produced by linear changes in these parameters over five transitional cohorts. The values of the parameters for each transitional cohort are shown in Table A4.6.1 along with the proportions ever married. These values provide a reference with which to compare the nuptiality parameters and proportions ever married used in the simulated fertility decline and shown in Table A4.6.2. No account is taken of declining C, final proportion ever married, in the simulation, but to some extent this is accounted for by the high values of k. It is seen immediately that the a_0 and k parameter ranges are very different. Lesthaeghe has a later age of start of first marriage, but a faster

Table A4.6.1: Nuptiality parameters and proportions ever married in Lesthaeghe's nuptiality transition

	Pre-	Transitional					Post-
Parameter	0	1	2	3	4	5	6
a_0	13.4	13.8	14.3	14.7	15.2	15.6	16.0
k	.400	.433	.466	.500	.533	.567	.600
C	.980	.967	.954	.940	.927	.914	.900
Age	Proportions ever married						
15-19	.474	.369	.269	.194	.124	.086	.052
20-24	.904	.858	.798	.722	.640	.552	.482
25-29	.975	.953	.933	.903	.870	.836	.795
30-34	.980	.967	.953	.936	.917	.898	.875
35-39	.980	.967	.954	.940	.927	.913	.896
40-44	.980	.967	.954	.940	.927	.914	.900
45-49	.980	.967	.954	.940	.927	.914	.900
Age	Proportions ever married for C = 1						
15-19	.484	.382	.282	.206	.134	.094	.058
20-24	.922	.887	.836	.768	.690	.604	.536
25-29	.995	.986	.978	.961	.939	.915	.883
30-34	1.000	1.000	.999	.996	.989	.982	.972
35-39	1.000	1.000	1.000	1.000	1.000	.999	.996
40-44	1.000	1.000	1.000	1.000	1.000	1.000	1.000
45-49	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table A4.6.2: Nuptiality parameters and proportions ever married by midpoint of each age group at each stage of the simulated fertility decline

Parameter	Stage of decline 1	2	3	4	5
a_o	10.0	10.5	11.0	11.5	12.0
k	0.6	0.7	0.8	0.9	1.0
Age	Proportions ever married				
10-14	.079	.032	.012	.004	.001
15-19	.653	.495	.353	.240	.157
20-24	.913	.836	.742	.637	.530
25-29	.979	.952	.909	.851	.786
30-34	.998	.987	.969	.943	.907
35-39	1.000	.999	.992	.978	.960
40-44	1.000	1.000	.999	.994	.984
45-49	1.000	1.000	1.000	1.000	.996

Table A4.6.2: Nuptiality parameters and proportions ever married by midpoint of each age group at each stage of the simulated fertility decline

	Stage of decline				
Parameter	1	2	3	4	5
a_o	10.0	10.5	11.0	11.5	12.0
k	0.6	0.7	0.8	0.9	1.0
Age	Proportions ever married				
10-14	.079	.032	.012	.004	.001
15-19	.653	.495	.353	.240	.157
20-24	.913	.836	.742	.637	.530
25-29	.979	.952	.909	.851	.786
30-34	.998	.987	.969	.943	.907
35-39	1.000	.999	.992	.978	.960
40-44	1.000	1.000	.999	.994	.984
45-49	1.000	1.000	1.000	1.000	.996

pace, than the simulated fertility decline values. In addition he has a wider range of a_o but a smaller range of k. The proportions ever married produced by these parameters can be seen and compared more clearly in Figure A4.6.1. Despite their different parameters and the absence of a changing C for the simulation, the two transitions overlap considerably. This is due to the faster pace of Lesthaeghe's nuptiality schedules compensating for their later start and to the simulations slower pace of marriage compensating for its absence of changing C levels. Though there are differences in the proportions, especially at very young ages and at very old ages for the later stages of the transition, the overall picture is one of agreement.

It would be expected, therefore that the two nuptiality transitions would produce similar changes in the pattern of age specific fertility. The differences at young ages might also be expected to be reduced in the fertility schedules because of the small numbers involved at such young ages. To make a direct comparison of the effects of the two nuptiality transitions on age specific fertility, Coale-Trussell model fertility schedules corresponding to the nuptiality parameters at each stage were calculated (in all cases for $m = 0$). The rates are shown in Table A4.6.3. Because the Coale-Trussell model is primarily concerned with patterns of fertility rather than level, C is assumed to be 1, and

Figure A4.6.1: Proportions ever married in the simulated fertility decline and in Lesthaeghe's nuptiality transition.

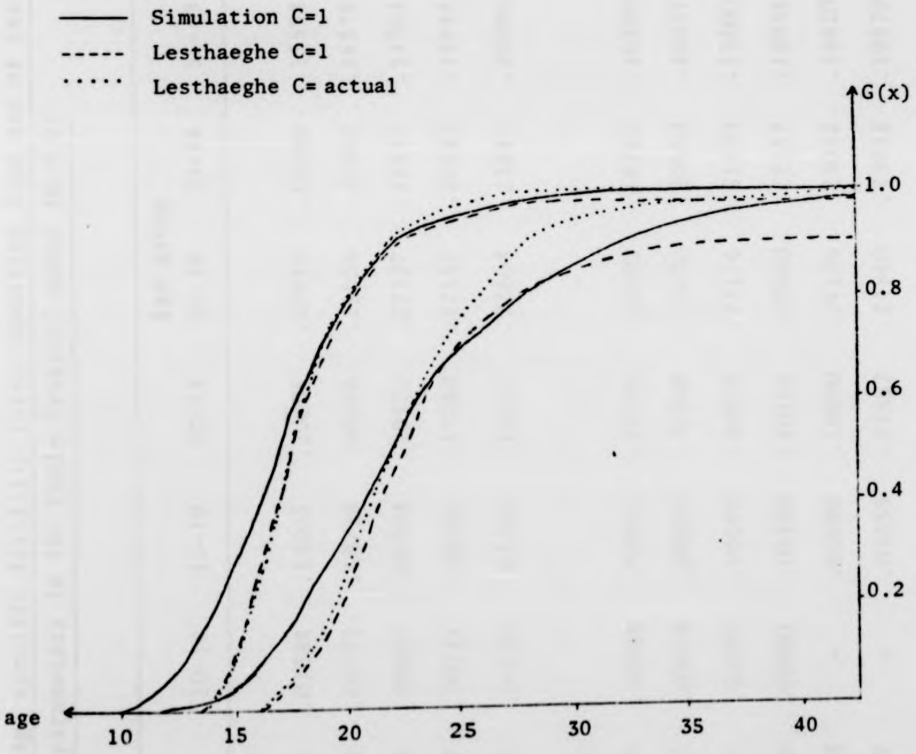


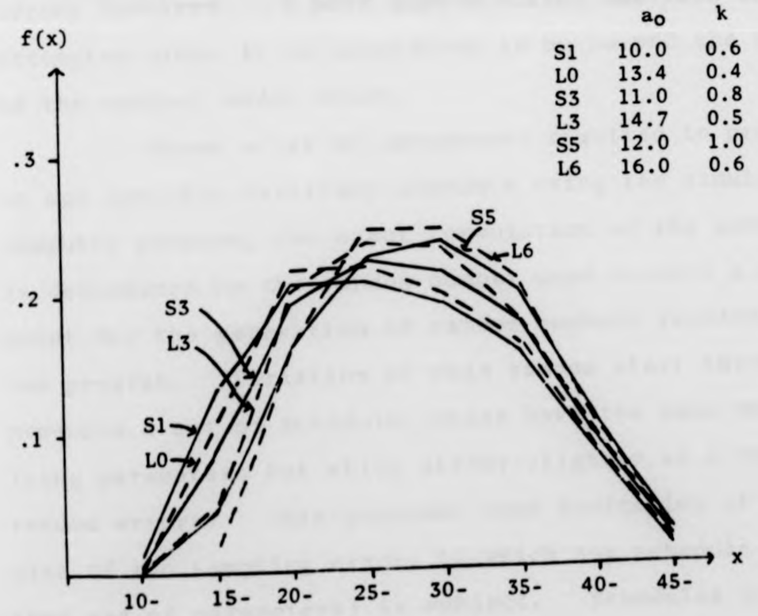
Table A4.6.3: Age specific fertility rates resulting from use of the transitional nuptiality parameters in the Coale-Trussell model (m = 0)

	Parameters		Age group							
No.	a_o	k	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49
	Simulation:									
1	10.0	0.60	.01384	.13005	.20609	.20916	.19206	.15598	.08072	.01210
2	10.5	0.70	.00772	.10728	.20096	.21656	.20282	.16579	.08597	.01290
3	11.0	0.80	.00417	.08504	.19122	.22227	.21423	.17703	.09218	.01385
4	11.5	0.90	.00221	.06532	.17760	.22557	.22563	.18943	.09928	.01496
5	12.0	1.00	.00115	.04898	.16127	.22603	.23647	.20266	.10721	.01623
	Lesthaeghe:									
0	13.4	0.40	.00098	.10631	.21730	.21902	.19921	.16128	.08340	.01250
1	13.8	0.43	.00033	.08854	.21538	.22470	.20550	.16652	.08612	.01291
2	14.3	0.47	.00006	.06698	.20939	.23176	.21430	.17400	.09002	.01349
3	14.7	0.50	.00001	.05199	.20192	.23680	.22157	.18036	.09335	.01399
4	15.2	0.53	-	.03690	.19050	.24196	.23032	.18820	.09750	.01462
5	15.6	0.57	-	.02555	.17552	.24540	.23933	.19678	.10210	.01532
6	16.0	0.60	-	.01758	.16067	.24732	.24732	.20472	.10641	.01597

variation in this parameter is not taken into account. This means that the two series of schedules are not directly comparable (becoming less comparable as the transition advances), because the effect of C (which is not accounted for) in Lesthaeghe's transition is to some extent described by k (which is accounted for) in the simulation transition. The effect on age specific fertility is one of slightly higher rates at ages greater than 25 for Lesthaeghe's series than would be obtained if C were taken into account. The extent of the difference in proportions ever married can be seen in Table A4.6.1 and Figure A4.6.1.

The results in Table A4.6.3 show general agreement in the effects of the simulation and Lesthaeghe's nuptiality transitions. It can be seen that the first, middle and last schedules of each series are reasonably alike (that is S1 and L0, S3 and L3, and S5 and L6, where S denotes simulation and L denotes Lesthaeghe). These 3 pairs of schedules are reproduced for comparison in Figure A4.6.2. As expected, Lesthaeghe's age specific fertility rates at ages 25 and over are slightly higher than those for the simulation (with those at ages less than 25 slightly lower by compensation). This suggests that had C been taken into account, these 3 pairs might have been even more closely matched. This close agreement between fertility schedules resulting from the two nuptiality transitions confirms the expectation that the differences that do exist between the nuptiality schedules are very much reduced in terms of fertility.

Figure A4.6.2: Age patterns of fertility for three pairs of Simulation and Lesthaeghe nuptiality parameters using the Coale-Trussell model.



APPENDIX 4.7

SAMPLING ERRORS

The comparisons between completed fertility rates, age specific fertility rates and ratios of rates involved in the development of a simulated fertility decline are many. Throughout, the presence of sampling errors should be borne in mind. The aim of this appendix is to give some idea as to the magnitude of the sampling errors involved. A more sophisticated analysis is not attempted since it is considered to be beyond the scope of the subject under study.

Given a set of parameters required to produce an age specific fertility schedule using the simulation computer program, the exact formulation of the schedule is determined by the random number used to mark a starting point for the generation of random numbers required by the program. Variation of this random start therefore produces a set of schedules which have the same underlying parameters but which differ slightly as a result of random errors. This provides some indication of the size of the sampling errors to which any schedule (with that set of parameters) is subject. Schedules produced from different sets of parameters will be subject to slightly different sizes of sampling errors, but this is not considered since the differences are small.

The age specific fertility rates produced from the modified Barrett simulation model, described in Chapter 4, with parameters DFS = 6, a_0 = 10 years, k = 0.6, E_1 = 0.7, E_2 = 0.9 and r = 1/6, but with ten different random starts, are shown in Table A4.7.1. Their means, standard errors and coefficients of variation are also presented: it is seen that the standard errors are small, amounting to less than 6% of the mean rates for ages 15 to 44. In the tails of the distribution, however, the small numbers involved render the rates far more variable especially at older ages, where the standard error is almost as large as the rate itself.

Ratios of the rates to the 20-24 rate are given in the lower half of Table A4.7.1. Again, the coefficients of variation show that these ratios are far more reliable for ages 15 to 44. They do, however, have slightly larger coefficients of variation than the rates.

Table A4.7.1: Age specific fertility rates generated for a set of parameters by changing the random start

Age	1	2	3	4	5	6	7	8	9	10	mean	s.e.	cv
10-14	.00713	.00751	.00871	.00822	.01095	.00718	.00910	.00850	.00993	.00776	.00850	.00123	14.5
15-19	.15011	.15461	.15239	.15816	.15589	.14511	.14926	.13669	.14530	.14653	.14940	.00633	4.2
20-24	.31640	.31499	.32079	.32008	.31439	.31455	.31409	.31834	.31645	.31598	.31661	.00238	0.8
25-29	.26961	.26712	.27429	.26062	.26521	.27919	.27227	.27072	.26818	.27237	.26996	.00513	1.9
30-34	.15829	.15793	.14995	.15661	.15433	.15508	.16115	.16324	.16242	.15842	.15774	.00402	2.5
35-39	.06853	.06744	.06531	.06837	.07108	.06844	.06614	.07224	.06794	.07068	.06862	.00217	3.2
40-44	.02592	.02533	.02212	.02485	.02433	.02678	.02415	.02567	.02396	.02344	.02465	.00135	5.5
45-49	.00365	.00507	.00627	.00308	.00365	.00368	.00350	.00460	.00582	.00465	.00440	.00107	24.3
50-54	.00035	-	.00017	.00025	.00017	-	.00035	-	-	.00017	.00015	.00014	97.1
level	5.749	5.724	5.742	5.836	5.754	5.713	5.715	5.648	5.843	5.801	5.7525	.05994	1.0
Ratios of rates to 20-24 rate													
10-14	.02253	.02384	.02715	.02568	.03483	.02283	.02668	.02670	.03138	.02456	.02662	.00386	14.5
15-19	.47443	.49084	.47505	.49413	.49585	.46133	.43760	.42938	.45916	.46373	.46815	.02267	4.8
20-24	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	-	-
25-29	.85212	.84803	.85505	.81423	.84357	.88759	.79824	.85041	.84746	.86198	.84587	.02455	2.9
30-34	.50028	.50138	.46744	.48928	.49089	.49302	.47246	.51279	.51326	.50136	.49422	.01518	3.1
35-39	.21659	.21410	.20359	.21360	.22609	.21758	.19391	.22693	.21469	.22369	.21508	.01015	4.7
40-44	.08192	.08042	.06895	.07764	.07739	.08514	.07080	.08064	.07571	.07418	.07728	.00503	6.5
45-49	.01154	.01610	.01955	.00962	.01161	.01170	.01026	.00144	.01839	.01472	.01249	.00517	41.4
50-54	.00111	-	.00053	.00078	.00054	-	.00103	-	-	.00054	.00045	.00044	96.3

APPENDIX 5.1

THE COMPUTER PROGRAMS USED IN FITTING THE TRANSFORMED GOMPERTZ MODEL

Programming for the estimation of the parameters of the transformed Gompertz model is divided into two programs, and is functional rather than elegant. The programs are reproduced at the end of this appendix.

Input is to the first program, TRUNK. The following parameters are required:

NRUN	number of sets of data to be analysed

For each set of data:

NN	first point to be included in analysis
M	last point to be included in analysis
L	= 0 if input is age specific fertility = 1 if input is cumulative fertility
LL	= 0 if Gompertz function to be fitted = 1 if transformed Gompertz function to be fitted
KK	= 0 if data are endpoints (at ages 15, 20, etc.) = 1 if data are midpoints (average parities)
NEWFIT	= 0 if fitting to cumulative fertility = 1 if fitting to transformed fertility (not used)
TF	= 0 if initial level estimate to be obtained from data = user supplied initial level estimate
MFIX	= 0 if equal weights to be used (except w(10-14)=0) = 1 if infinite weight to last point

Title card

Data

This information is partially processed in TRUNK and is written onto tape in the form required by the minimisation program, MINUIT (James and Roos, 1971), which is external to the programming described here. The second program, TRUNKIT, calls MINUIT as required, and both programs read the prepared data from tape.

The objective function is calculated at each iteration by the subroutine FCN within TRUNKIT. This subroutine performs the main part of the fitting procedure by providing the specific information for the general minimisation carried out by MINUIT. FCN also produces the output, an example of which is reproduced.

```
      PROGRAM [illegible](INPUT,OUTPUT,TAPE1)
      DIMENSION F([illegible]),FF( )
C                 NRUNS=NUMBER OF RUNS.
C                 FIT IS BASED ON NITH TO MTH POINTS WHERE M≤8
C                 CHOOSE L VALUE FOR DATA REQUIREMENTS
C                 L=0 IF ASFR
C                 L=1 IF CUMULATIVE ASFR
C                 CHOOSE LL VALUE FOR REQUIRED FUNCTION
C                 LL=0 FOR F(X)=F*A**B**Y
C                 LL=1 FOR F(X)=F*A**B**YS(X)
C                 CHOOSE KK VALUE FOR REQUIRED AGE POINTS
C                 KK=0 FOR STANDARD AT POINTS 15, 20,...
C                 KK=1 FOR STANDARD AT POINTS 12.5, 17.5,...
C                 CHOOSE NEWFIT VALUES FOR FORM OF DATA
C                 NEWFIT=0 FOR CUMULATIVE FERTILITY, F(X)
C                 NEWFIT=1 FOR TRANSFORMED CUMULATIVE FERTILITY, Y(X)
C                 TF IS USER SUPPLIED INITIAL ESTIMATE OF THE LEVEL
C                 OF FERTILITY (OPTIONAL)
C                 IF BLANK OR ZERO, TF IS ESTIMATED FROM THE DATA
C                 MFIX=0 EQUAL WEIGHTS TO POINTS, FIRST POINT NOT
C                 INCLUDED
C                 MFIX=1 INFINITE WEIGHT TO LAST REPORTED POINT
      N=0
      READ 99, NRUN
 99   FORMAT(I2)
      DO 2 I=1,NRUN
      READ 200, N,M,L,LL,KK,NEWFIT,TF,MFIX
 200  FORMAT(6I2,F5.2,2X,I1)
      READ 100,[illegible]
 100  FORMAT([illegible]
      IF(L.EQ.1) GO TO 4
      READ 98,(F(I),I=1,M)
 98   FORMAT(8F10.5)
C                 CALCULATE CUMULATIVE FERTILITY
      FF(1)=F(1)
      DO 1 I=2,M
      FF(I)=F(I)+FF(I-1)
 1    CONTINUE
      GO TO 3
 4    CONTINUE
      READ 98,(FF(I),I=1,M)
 3    CONTINUE
      NM=N+1
C                 CALCULATE INITIAL PARAMETER ESTIMATES FROM DATA
      NRANGE=M-NM
      IF(NRANGE.LT.1) STOP
      IF(NRANGE.EQ.2.OR.NRANGE.EQ.3) GO TO 400
      IF(NRANGE.EQ.4.OR.NRANGE.EQ.5) GO TO 420
      IF(NRANGE.EQ.6.OR.NRANGE.EQ.7) GO TO 440
      IF(NRANGE.GE.8) STOP
 400  CONTINUE
      INT=5
      K0=M-2
      K1=M-1
      K2=M
      GO TO 460
 420  INT=10
      K0=M-4
      K1=M-2
      K2=M
      GO TO 460
 440  INT=15
      K0=M-6
      K1=M-3
      K2=M
 460  CONTINUE
      D21=ALOG(FF(K2))-ALOG(FF(K1))
      D10=ALOG(FF(K1))-ALOG(FF(K0))
      RINT=D21/D10
      B=EXP(ALOG(RINT)/INT)
      ZLNA=D10**2/(D21-D10)
      ZA=EXP(ZLNA)
      A=ZA**(-B**(10.7/2-(KK*5.0+10.0-2.5*KK)))
      IF(TF   .NE.0.0) GO TO 600
      ZLNTF=ALOG(FF(K0))-ZLNA
      TF   =EXP(ZLNTF)
 600  CONTINUE
      C=0.041
      D=0.7[illegible]
      BETA=EXP((-1.0*ALOG(C))**(-1.0*ALOG(M)/ALOG(D)))
      [illegible]=(-1.0*ALOG(M)/ALOG(D))
      ALPHA=-1.0*ALOG(-1.0*ALOG(P))
      BETA=-1.0*ALOG(Q)
```

```
C                  WRITE DATA ONTO TAPE1
         WRITE(1,100)
         IF([illegible]) GO TO 5
         TF=[illegible](4)
         GO TO 6
  5      CONTINUE
         TF=[illegible](7)
  6      CONTINUE
         WRITE(1,101) TF,TFMIN
                               1TF',8X,F10.5,'0.1',7X,F10.5,'12.0')
 101     FORMAT('
         IF([illegible].1) GO TO 500
         WRITE(1,102) A
         WRITE(1,103) B
         GO TO 501
 500     CONTINUE
         IF([illegible]IT.EQ.1) GO TO  50
         WRITE(1,102) P
                               2P',9X,F10.5,'0.05       0.0        1.0')
 102     FORMAT('
         WRITE(1,103) Q
                               3Q',9X,F10.5,'0.05       0.0        1.0')
 103     FORMAT('
         GO TO 501
 50      CONTINUE
         WRITE(1,120) ALPHA
                               2ALPHA',5X,F10.5,'0.05        -1.0        1.0')
 120     FORMAT('
         WRITE(1,121) BETA
                               3BETA',5X,F10.5,'0.05        0.0        2.0')
 121     FORMAT('
 501     CONTINUE
         WRITE(1,104)
 104     FORMAT(1X)
C                  DATA FOR FCN
         WRITE(1,99) NFIX
         WRITE(1,300) N,M,LL,KK,NEWFIT
 300     FORMAT(5I2)
         WRITE(1,[illegible])
         WRITE(1,301) (FF(I),I=1,M)
 301     FORMAT(8F10.5)
C
         WRITE(1,105)
 105     FORMAT('MINIMIZE')
         IF([illegible]) GO TO 106
         CONTINUE
         WRITE(1,107)
 107     FORMAT('[illegible]')
         GO TO 108
 106     WRITE(1,109)
 109     FORMAT('EXIT')
 108     CONTINUE
 2       CONTINUE
         STOP
         END
```

```
      PROGRAM [illegible]FIT(INPUT,OUTPUT,TAPE1,TAPE2=OUTPUT)
      CALL [illegible]
      [illegible]
      [illegible]

      SUBROUTINE FCN(NPAR,G,H,X,IFLAG)
      DIMENSION LABEL(8),FF(8),FITFF(8),X(3),YS(7),DFF(8),PDFF(8)
      DIMENSION FITY(8),W(8),W(8),Y(8),Z(7),YSM(9),R(8),RP(8)
      DATA LABEL/'10-14','15-19','20-24','25-29','30-34','35-39',
     *'40-44','45-49'/
      DATA YS/-1.773[illegible],-.69130,0.02564,0.7,1.47572,2.62602,4.80970/
      DATA YSM/-2.67323,-1.07[illegible],-.311279,.353802,
     *1.[illegible],1.[illegible],3.413020,5.055691/
      IF(IFLAG.NE.1) GO TO 3
C                         READ DATA FROM TAPE1
      READ(1,[illegible]) NFIT
      READ(1,[illegible]) M,LL,KK,NEWFIT
 99   FORMAT(5I2)
      READ(1,98)
 98   FORMAT([illegible]
      READ(1,97) (FF(I),I=1,M)
 97   FORMAT(8F10.5)
 3    CONTINUE
      W=0.0
      TF=X(1)
      A=X(2)
      B=X(3)
      IF(NEWFIT.EQ.1) GO TO 50
      IF(LL.EQ.1) GO TO 10
C                 CALCULATION OF FIT VALUES FOR ORDINARY
C                 GOMPERTZ FUNCTION
      DO 9 I=1,8
      Z(I)=I[illegible]*5.0 -2.5*KK
 9    CONTINUE
      DO 11 I=1,M
      FITFF(I)=TF*A**(B**(Z(I)-16.732))
 11   CONTINUE
      GO TO 12
 10   CONTINUE
C                 CALCULATION OF FIT VALUES FOR TRANSFORMED GOMPERTZ
C                 WHEN FITTING IN F
      IF(KK.EQ.1) GO TO 2
      IF(M.EQ.8) M1=7
      IF(M.LT.8) M1=M
      DO 1 I=1,M1
      FITFF(I)=TF*A**(B**YS(I))
 1    CONTINUE
      IF(M.EQ.8) FITFF(8)=TF
      GO TO 4
 2    CONTINUE
      DO 15 I=1,M
      FITFF(I)=TF*A**(B**YSM(I))
 15   CONTINUE
 4    CONTINUE
 12   CONTINUE
      DO 17 I=1,M
      DFF(I)=FF(I)-FITFF(I)
      PDFF(I)=DFF(I)/FF(I)
      W(I)=1.0
 17   CONTINUE
      IF(NFIT.EQ.1) W(M)=100000000.
      GO TO 16
C                 CALCULATION OF FIT VALUES WHEN FITTING
C                 TRANSFORMED MODEL IN TRANSFORMED SCALE
 50   CONTINUE
      IF(KK.EQ.1) GO TO 30
      IF(M.EQ.8) M1=7
      IF(M.LT.8) M1=M
      DO 31 I=1,M1
      FITY(I)=A+B*YS(I)
      FITFF(I)=(EXP(-1.0*EXP(-1.0*FITY(I))))*TF
 31   CONTINUE
      IF(M.EQ.8) FITFF(8)=TF
      GO TO 32
 30   CONTINUE
      DO 51 I=1,M
      FITY(I)=A+B*YSM(I)
      FITFF(I)=(EXP(-1.0*EXP(-1.0*FITY(I))) )*TF
 51   CONTINUE
 32   CONTINUE
      DO 66 I=1,M
      IF(FF(I).EQ.0.0) GO TO 66
      IF(FF(I).EQ.TF) GO TO 68
      Y(I)=-1.0*ALOG(ALOG(TF)-ALOG(FF(I)))
      DFF(I)=Y(I)-FITY(I)
      PDFF(I)=[illegible]
      IF(FITFF(I).EQ.0.0) GO TO 68
      R(I)=FITFF(I)/TF
      W(I)=(R(I)*ALOG(R(I)))**2
      GO TO [illegible]
 68   CONTINUE
      DFF(I)=0.0
      W(I)=0.0
 66   CONTINUE
 16   CONTINUE
```

```
      PROGRAM [illegible](INPUT,OUTPUT,TAPE1,TAPE2=OUTPUT)
      [illegible]
      [illegible]
      [illegible]

      SUBROUTINE [illegible](M,[illegible],X,IFLAG)
      DIMENSION [illegible](8),FF(8),FITFF(8),X(3),YS(7),DFF(8),PDFF(8)
      DIMENSION FITY(8),[illegible](8),W(8),Y(8),Z(7),YSM(8),R(8),RPW(8)
      DATA [illegible]/'[illegible]-14','15-19','20-24','25-29','30-34','35-39',
     *   '[illegible]','[illegible]' /
      DATA YS/-1.77[illegible],-.69130,0.02564,0.7,1.47872,2.62602,4.80970/
      DATA YSM/-5.02323[illegible],-1.07482[illegible],-.311875,.353802,
     *   1.[illegible], 1.93[illegible], 3.413020, 6.055691 /
      IF(IFLAG.NE.1) GO TO 3
C                   READ DATA FROM TAPE1
      READ(1,[illegible]) [illegible]
      READ(1,[illegible]) N,M,LL,K,NEWFIT
 [illegible] FORMAT(5I2)
      READ(1,[illegible])
 [illegible] FORMAT([illegible]
      READ(1,57) (FF(I),I=1,M)
 57   FORMAT([illegible]F10.5)
 3    CONTINUE
      W=0.0
      TF=X(1)
      A=X(2)
      B=X(3)
      IF(NEWFIT.EQ.1) GO TO 50
      IF(LL.EQ.1) GO TO 10
C                   CALCULATION OF FIT VALUES FOR ORDINARY
C                   GOMPERTZ FUNCTION
      DO 9 I=1,8
      Z(I)=[illegible]*5.0 -2.5*KK
 9    CONTINUE
      DO 11 I=1,M
      FITFF(I)=TF*A**(B**(Z(I)-16.732))
 11   CONTINUE
      GO TO 12
 10   CONTINUE
C                   CALCULATION OF FIT VALUES FOR TRANSFORMED GOMPERTZ
C                   WHEN FITTING IN F
      IF(K.EQ.1) GO TO 2
      IF(M.EQ.8) M1=7
      IF(M.LT.8) M1=M
      DO 1 I=1,M1
      FITFF(I)=TF*A**(B**YS(I))
 1    CONTINUE
      IF(M.EQ.8) FITFF(8)=TF
      GO TO 4
 2    CONTINUE
      DO 15 I=1,M
      FITFF(I)=TF*A**(B**YSM(I))
 15   CONTINUE
 4    CONTINUE
 12   CONTINUE
      DO 17 I=1,M
      DFF(I)=FF(I)-FITFF(I)
      PDFF(I)=DFF(I)/FF(I)
      W(I)=1.0
 17   CONTINUE
      IF(NEFIT.EQ.1) W(M)=100000000.
      GO TO 16
C                   CALCULATION OF FIT VALUES WHEN FITTING
C                   TRANSFORMED MODEL IN TRANSFORMED SCALE
 50   CONTINUE
      IF(K.EQ.1) GO TO 30
      IF(M.EQ.8) M1=7
      IF(M.LT.8) M1=M
      DO 31 I=1,M1
      FITY(I)=A+B*YS(I)
      FITFF(I)=(EXP(-1.0*EXP(-1.0*FITY(I))))*TF
 31   CONTINUE
      IF(M.EQ.8) FITFF(8)=TF
      GO TO 32
 30   CONTINUE
      DO 51 I=1,M
      FITY(I)=A+B*YSM(I)
      FITFF(I)=(EXP(-1.0*EXP(-1.0*FITY(I))) )*TF
 51   CONTINUE
 32   CONTINUE
      DO 56 I=1,M
      IF(FF(I).[illegible]) GO TO 66
      IF(FF(I).[illegible]TF) GO TO 66
      Y(I)=-1.0*ALOG(ALOG(TF)-ALOG(FF(I)))
      DFF(I)=Y(I)-FITY(I)
      PDFF(I)=[illegible]
      IF(FITFF(I).EQ.0.0) GO TO 66
      R(I)=FITFF(I)/TF
      W(I)=(R(I)*ALOG(R(I)))**2
      GO TO 56
 66   CONTINUE
      DFF(I)=0.0
      W(I)=0.0
 56   [illegible]
 16   CONTINUE
```

```
C                     CALCULATE OBJECTIVE FUNCTION
      DO 18 I=N,M
      W(I)=(DFF(I)**2) * RRW(I)
      R=R+W(I)
 18   CONTINUE
      IF(IFLAG.EQ.3) GO TO 5
      RETURN
 5    CONTINUE
      DO 81 I=N,M
      IF(I.GT.3) GO TO 81
      PP(I)=P(I)/PP(3)
 81   CONTINUE
C                     PRINT OUT RESULTS OF FIT IN CUMULATIVE FERTILITY
      PRINT 400
 400  FORMAT(1H1)
      PRINT 98
      PRINT 100,N,M
 100  FORMAT(//1H0,'FIT IN CUMULATIVE FERTILITY BASED ON POINTS',I3,
     *    ' TO',I3)
C
      IF(NEWFIT.EQ.1) GO TO 55
      IF(LL.EQ.1) GO TO 13
C                     OUTPUT FOR ORDINARY GOMPERTZ
      PRINT 300
 300  FORMAT(//' FITTING F(X)=F*G**H**X')
      PRINT 301,TF,A,B,R
 301  FORMAT(1H0,'TF=',F8.5,' G=',F8.5,' H=',F8.5,' OBJ FN (R) =',F8.5)
      GO TO 14
 13   CONTINUE
C                     OUTPUT FOR TRANSFORMED GOMPERTZ....FITTING IN F
      PRINT 106
 106  FORMAT(//' FITTING F(X)=F*A**B**YS(X) ')
      PRINT 101,TF,A,B,R
 101  FORMAT(1H0,'TF=',F8.5,' A=',F8.5,' B=',F8.5,' OBJ FN (R) =',F8.5)
      GO TO 14
C                     OUTPUT FOR TRANSFORMED GOMPERTZ...FITTING IN Y
 55   CONTINUE
      IF(KK.EQ.1) GO TO 40
      PRINT 41
 41   FORMAT(//' FITTING F(X)=F*A**B**YS(X)'/
     *    '        NEWFIT -LN(LN(TF)-LN(F(X)))=ALPHA+BETA*YS(X)')
      GO TO 42
 40   CONTINUE
      PRINT 56
 56   FORMAT(//' FITTING F(X)=F*A**B**YSM(X)'/
     *    '        NEWFIT  -LN(LN(TF)-LN(F(X)))=ALPHA+BETA*YSM(X)')
 42   CONTINUE
      PRINT 52,TF,A,B,R
 52   FORMAT(1H0,'TF=',F8.5,' ALPHA=',F8.5,' BETA=',F8.5,
     *    ' OBJ FN=',F10.8)
      GO TO 53
C
 14   CONTINUE
      PRINT 102
 102  FORMAT(1H0,' AGE       FF          FITFF      DIFF      DIFF/FF
     *REL WGT')
      DO 103 I=1,M
      PRINT 60 ,LABEL(I),FF(I),FITFF(I),DFF(I),PDFF(I),RRW(I)
 60   FORMAT(1H ,A5,4F10.5,F20.10)
 103  CONTINUE
      K=M+1
      IF(K.EQ.9) GO TO 8
      IF(KK.EQ.1) GO TO 19
      IF(K.EQ.8) GO TO 22
      DO 6 I=K,7
      FITFF(I)=TF*A**(B** YS(I))
 6    CONTINUE
 22   CONTINUE
      FITFF(8)=TF
      GO TO 20
 19   CONTINUE
      DO 21 I=K,8
      FITFF(I)=TF*A**(B**YSM(I))
 21   CONTINUE
 20   CONTINUE
      DO 7 I=K,8
      PRINT 105,LABEL(I),FITFF(I)
 105  FORMAT(1H ,A5,10X,F10.5)
 7    CONTINUE
 8    CONTINUE
C                     CALCULATE IMPLIED FIT IN LOG(-LOG) TRANSFORM
      ALPHA=ALOG(-1.0*ALOG(A)) *(-1.0)
      BETA=ALOG(B) *(-1.0)
      PRINT 201
 201  FORMAT(//1H0,'IMPLIED FIT IN LOG(-LOG) TRANSFORM')
      PRINT 202,ALPHA,BETA
 202  FORMAT(1H0,'ALPHA=',F8.5/1H ,'BETA =',F8.5)
C
      GO TO 70
 53   CONTINUE
      IF(M.EQ.8) GO TO 62
      K=M+1
      IF(KK.EQ.1) GO TO 43
      IF(K.EQ.8) GO TO 45
      DO 44 I=K,7
      FITY(I)=A+B*YS(I)
      FITFF(I)=EXP(-1.0*EXP(-1.0*FITY(I)))*TF
 44   CONTINUE
 45   CONTINUE
      FITFF(8)=TF
      FITY(8)=99.99999
```

```
      [illegible]
 43   CONTINUE
      DO 52 I=K,M
      FITY(I)=A+B*XSUM(I)
      FITFF(I)=EXP(-1.0*EXP(-1.0*FITY(I))) * TF
 52   CONTINUE
 42   CONTINUE
 44   CONTINUE
      PRINT 59
 59   FORMAT(1H0,'AGE          Y          FITY       DIFF       W         REL
     *[illegible]')
      DO 49 I=1,M
      PRINT 60,LABEL(I),Y(I),FITY(I),DFF(I),W(I),RRW(I)
 49   CONTINUE
      IF(K.EQ.M) GO TO 72
      DO 71 I=K,M
      PRINT 60, LABEL(I),FITY(I)
 71   CONTINUE
 72   CONTINUE
      AA=EXP(-1.0*EXP(-1.0*A))
      BB=EXP(-1.0*B)
      PRINT 63
 63   FORMAT(//1H0,' IMPLIED FIT IN F')
      PRINT 64,AA,BB
 64   FORMAT(1H0,'P=',F8.5,' Q=',F8.5)
      PRINT 162
      DO 65 I=1,M
      DFF(I)=FF(I)-FITFF(I)
      PDFF(I)=DFF(I)/FF(I)
      PRINT 60,LABEL(I),FF(I),FITFF(I),DFF(I),PDFF(I),RRW(I)
 65   CONTINUE
      IF(K.EQ.M) GO TO 73
      DO 67 I=K,M
      PRINT 69,LABEL(I),FITFF(I)
 69   FORMAT(1H ,A5,10X,F10.5)
 67   CONTINUE
 73   CONTINUE
 70   CONTINUE
      RETURN
      END
```

```
  BANGLADESH 35-39 COHORT (CUMULATED TO 12.5,17.5...

FIT IN CUMULATIVE FERTILITY BASED ON POINTS  2 TO  6

FITTING F(X)=F*A**B**YS(X)
TF= 7.86538 A=  .41046 B=  .42644 OBJ FN (R) =  .07593
  AGE        FF         FITFF      DIFF      DIFF/FF        REL WGT
 10-14     .04300      .04507   -.00207   -.04810               -R
 15-19     .83950      .85243   -.01293   -.01540         .0102231451
 20-24    2.33800     2.46590   -.12790   -.05470        1.0000000000
 25-29    4.05300     4.06623   -.01323   -.00326         .0106948981
 30-34    5.71150     5.46812    .24338    .04261        3.6211939950
 35-39    6.63550     6.63550   -.00000   -.00000         .0000015605
 40-44                7.46668
 45-49                7.82417

 IMPLIED FIT IN LOG(-LOG) TRANSFORM

 ALPHA=  .11600
 BETA =  .84700
```

```
                               ***********************
                               *   D506   MINUIT     *
                               *   VERSION 11.76     *
                               *   DATA BLOCK NO.  2 *
                               ***********************
 ******************************************************************************
      BANGLADESH  [illegible]  CONST  (CUMULATED TO 12.5 [illegible]              TIME  5.631
 ******************************************************************************
 ******************************************************************************
   1  TF               8.30750      .100000        6.63550      12.6000
   2  A                 .394420     .500000E-01    0.           1.00000
   3  B                 .419750     .500000E-01    0.           1.00000
 ******************************************************************************
FIRST ENTRY TO FCN
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .1126172E+06       1       5.68    0.            1    1  TF        .83075E+01   .10000E+00   -.35516E+00    .40262E-01
                                                  2    2  A         .39442E+00   .50000E-01   -.23327E+00    .10300E+00
                                                  3    3  B         .41975E+00   .50000E-01   -.16120E+00    .10150E+00
**********
***  1****MINIMIZE
**********
START SIMPLEX MINIMIZATION                       CONVERGENCE CRITERION -- ESTIMATED DISTANCE TO MINIMUM (EDM) .LT.   .10E+00
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .5074000E+04      12       5.70    .11E+06       1    1  TF        .79132E+01   .63216E-01   -.54552E+00    .20436E+00
                                                  2    2  A         .43505E+00   .35073E-01   -.13027E+00    .10300E+00
                                                  3    3  B         .41975E+00   .35391E-01   -.16120E+00    .10150E+00
SIMPLEX MINIMIZATION HAS CONVERGED
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .8394430E-01      69       5.76    .50E-01       1    1  TF        .78450E+01   .20717E-02   -.57234E+00    .19237E-03
                                                  2    2  A         .41304E+00   .44473E-03   -.17471E+00    .10112E-03
                                                  3    3  B         .42447E+00   .40781E-03   -.14051E+00    .17202E-03
                                                 ERRORS CORRESPOND TO FUNCTION CHANGE OF     1.0000
START MIGRAD MINIMIZATION.                       CONVERGENCE CRITERIA -- ESTIMATED DISTANCE TO MINIMUM (EDM) .LT.   .10E-05
                                                 OR EDM .LT.  .10E+00 AND FRACTIONAL CHANGE IN VARIANCE MATRIX .LT.  .40E-01
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .8394439E-01      75       5.79    .17E-01       1    1  TF        .78450E+01   .11553E-02   -.57234E+00    .18376E-04
                                                  2    2  A         .41304E+00   .32373E-03   -.17471E+00    .18066E-04
                                                  3    3  B         .42447E+00   .19513E-03   -.14051E+00    .16475E-04
MIGRAD FAILS TO FIND IMPROVEMENT
MIGRAD TERMINATED WITHOUT CONVERGENCE
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .7620234E-01     105       5.81    .67E-02       1    1  TF        .78454E+01   .47551E-02   -.57719E+00   -.56117E-04
                                                  2    2  A         .41000E+00   .37243E-01   -.18004E+00    .18990E-02
                                                  3    3  B         .42444E+00   .21667E-01   -.14361E+00    .11005E-02
                                                 ERRORS CORRESPOND TO FUNCTION CHANGE OF     1.0000
START SIMPLEX MINIMIZATION                       CONVERGENCE CRITERION -- ESTIMATED DISTANCE TO MINIMUM (EDM) .LT.   .10E+00
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .7620234E-01     114       5.83    .30E-03       1    1  TF        .78454E+01   .33629E-02   -.57719E+00    .33739E-03
                                                  2    2  A         .41000E+00   .26335E-01   -.18004E+00    .12126E-01
                                                  3    3  B         .42444E+00   .15324E-01   -.14361E+00    .70070E-02
SIMPLEX MINIMIZATION HAS CONVERGED
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .7620234E-01     134       5.85    .24E-01       1    1  TF        .78454E+01   .27424E-02   -.57719E+00    .22317E-03
                                                  2    2  A         .41000E+00   .10625E-01   -.18004E+00    .30051E-02
                                                  3    3  B         .42444E+00   .65471E-02   -.14361E+00    .17920E-02
                                                 ERRORS CORRESPOND TO FUNCTION CHANGE OF     1.0000
START MIGRAD MINIMIZATION.                       CONVERGENCE CRITERIA -- ESTIMATED DISTANCE TO MINIMUM (EDM) .LT.   .10E-05
                                                 OR EDM .LT.  .10E+00 AND FRACTIONAL CHANGE IN VARIANCE MATRIX .LT.  .40E-01
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .7620234E-01     140       5.87    .10E-02       1    1  TF        .78454E+01   .11433E-02   -.57719E+00    .26766E-05
                                                  2    2  A         .41000E+00   .32349E-03   -.18004E+00    .66017E-03
                                                  3    3  B         .42444E+00   .19439E-03   -.14361E+00    .20442E-03
MIGRAD MINIMIZATION HAS CONVERGED
  FCN VALUE       CALLS     TIME      EDM      INT.EXT. PARAMETER     VALUE        ERROR       INTERN.VALUE   INT.STEP SIZE
 .7543449E-01     192       5.89    .98E-07       1    1  TF        .78454E+01   .10447E-02   -.57719E+00   -.58057E-07
                                                  2    2  A         .41000E+00   .29531E-03   -.18009E+00   -.43559E-06
                                                  3    3  B         .42444E+00   .17133E-03   -.14361E+00   -.30270E-06
                                                 ERRORS CORRESPOND TO FUNCTION CHANGE OF     1.0000
          INTERNAL COVARIANCE MATRIX             LAST FRACTIONAL CHANGE WAS    .124729
          CORRELATION COEFFICIENTS
       INT.  1      2
         2 -.204
         3  .207   .204
                                                                         GLOBAL CORRELATION
                                                          PARAMETER         COEFFICIENT
                                                           1  TF              .22776
                                                           2  A               .22602
                                                           3  B               .22776
**********
***  2****END
**********
CALL TO FCN WITH IFLAG = 3
```

APPENDIX 5.2

CALCULATION OF THE INITIAL PARAMETER ESTIMATES

Initial estimates of the parameters of the transformed Gompertz model are required as a starting point for the iterative fitting procedure. These rough values are provided internally to the computer program from knowledge of the data and of the standard. This procedure avoids the need for external (user provided) estimates, thereby relieving the user of the need to assess every set of data. It also gives a fairly accurate starting point for each set of data, thus minimising the number of iterations and the likelihood of finding possible local minima.

The procedure adopted here for estimating initial values of F, P and Q in the transformed Gompertz model is based on the assumption that the ordinary Gompertz model fits observed and standard fertility well enough for this purpose. Under this assumption, the two sets of Gompertz parameters can be used to estimate the transformed Gompertz parameters. If observed fertility can be represented by

$$F(x) = F\, A^{B^{x-x_0}}$$

and standard fertility by

$$F_s(x) = F_s\, C^{D^{x-x_o}}$$

where $F_s = 1$, $F > 0$ and $0 < A, B, C, D < 1$, then taking the double logarithm transforms (as in Chapter 2) gives

$$Y(x) = -\ln(-\ln A) - (x - x_o)\ln B$$

$$Y_s(x) = -\ln(-\ln C) - (x - x_o)\ln D$$

Writing $a = -\ln(-\ln A)$

$c = -\ln(-\ln C)$

$b = -\ln B$

$d = -\ln D$

where $-\infty < a,c < \infty$ and $0 < b,d < \infty$, gives

$$Y(x) = a + b(x - x_o)$$

$$Y_s(x) = c + d(x - x_o)$$

Hence

$$x - x_o = \frac{Y(x) - a}{b} = \frac{Y_s(x) - c}{d}$$

and thus $Y(x) = a - \frac{cb}{d} + \frac{b}{d}Y_s(x)$ AS.2.1

which is a linear relationship between observed and standard transformed fertility. Recalling from Chapter 2 that the transformed Gompertz model is described by

$$Y(x) = \alpha + \beta\, Y_s(x)$$

$$F_s(x) = F_s\, C^{D^{x-x_o}}$$

where $F_s = 1$, $F > 0$ and $0 < A, B, C, D < 1$, then taking the double logarithm transforms (as in Chapter 2) gives

$$Y(x) = -\ln(-\ln A) - (x - x_o)\ln B$$

$$Y_s(x) = -\ln(-\ln C) - (x - x_o)\ln D$$

Writing $a = -\ln(-\ln A)$
$c = -\ln(-\ln C)$
$b = -\ln B$
$d = -\ln D$
where $-\infty < a,c < \infty$ and $0 < b,d < \infty$, gives

$$Y(x) = a + b(x - x_o)$$

$$Y_s(x) = c + d(x - x_o)$$

Hence

$$x - x_o = \frac{Y(x) - a}{b} = \frac{Y_s(x) - c}{d}$$

and thus $Y(x) = a - \frac{cb}{d} + \frac{b}{d}Y_s(x)$ A5.2.1

which is a linear relationship between observed and standard transformed fertility. Recalling from Chapter 2 that the transformed Gompertz model is described by

$$Y(x) = \alpha + \beta\, Y_s(x)$$

it is seen that under the assumption that the Gompertz fit is adequate

$$\alpha = a - c\frac{b}{d} \quad \text{and } \beta = \frac{b}{d} .$$

Since $\alpha = -\ln(-\ln P)$ and $\beta = -\ln Q$, it follows that

$$P = A^{(-\ln C)^{-\ln B/\ln D}} \quad \text{and} \quad Q = B^{-1/\ln D} \qquad \text{A5.2.2}$$

It is thus possible to estimate P and Q from estimates of A, B, C and D.

Estimates of C and D for the standard have already been derived in Appendix 3.3 where the value of the origin, x_0, giving the best fit of the Gompertz model is also estimated. These estimates are $C = 0.0481$, $D = 0.8748$ and $x_0 = 16.732$.

Estimation of A and B for observed fertility, and of observed completed fertility, F, is done by the method of selected points, described in Appendix 5.3. The age range for this calculation is taken to be as wide and late as the data allow, given that the three selected points need to be equidistant. Table A5.2.1 shows which datapoints are used when complete and less than complete data are to be analysed. For midpoint data, the ages shown are not the exact ages to which the data, which are average parities, refer. This means that the points are not exactly equidistant, though the error involved is small especially for the range of ages usually employed

Table A5.2.1: Selected points used in the estimation of the ordinary Gompertz parameters

Points included	Selected points		
	Endpoint data		
15 to 50	20	35	50
15 to 45	15	30	45
15 to 40	20	30	40
15 to 35	15	25	35
15 to 30	20	25	30
20 to 50	20	35	50
20 to 45	25	35	45
20 to 40	20	30	40
20 to 35	25	30	35
25 to 50	30	40	50
25 to 45	25	35	45
25 to 40	30	35	40
	Midpoint data*		
12.5 to 47.5	17.5	32.5	47.5
12.5 to 42.5	12.5	27.5	42.5
12.5 to 37.5	17.5	27.5	37.5
12.5 to 32.5	12.5	22.5	32.5
17.5 to 47.5	17.5	32.5	47.5
17.5 to 42.5	22.5	32.5	42.5
17.5 to 37.5	17.5	27.5	37.5
17.5 to 32.5	22.5	27.5	32.5
22.5 to 47.5	27.5	37.5	47.5
22.5 to 42.5	22.5	32.5	42.5
22.5 to 37.5	27.5	32.5	37.5

* These ages are not the ages to which the data actually refer.

and is not important for the purposes of providing initial estimates.

The estimate of A (A_1 say) is that obtained when the origin is equal to the age of the first selected point and needs to be adjusted to an origin of 16.732 to be compatible with the standard value. (B is unaffected by changes in x_o.) Hence the estimate of A used in equation A5.2.2 to obtain an estimate of P is

$$A = A_1 B^{16.732 - x_o'}$$

where x_o' is the age of the first selected point on which A_1 is calculated.

For the analyses reported in Chapters 5 and 6 the parameters F, P and Q are estimated by the above procedure and used as the initial parameter values in the iterative estimation.

and is not important for the purposes of providing initial estimates.

The estimate of A (A_1 say) is that obtained when the origin is equal to the age of the first selected point and needs to be adjusted to an origin of 16.732 to be compatible with the standard value. (B is unaffected by changes in x_o.) Hence the estimate of A used in equation A5.2.2 to obtain an estimate of P is

$$A = A_1 B^{16.732 - x_o'}$$

where x_o' is the age of the first selected point on which A_1 is calculated.

For the analyses reported in Chapters 5 and 6 the parameters F, P and Q are estimated by the above procedure and used as the initial parameter values in the iterative estimation.

APPENDIX 5.3

ESTIMATION OF THE GOMPERTZ PARAMETERS BY THE METHOD OF SELECTED POINTS

The Gompertz parameters can be estimated from three equidistant points by the method of selected points. This method has been used by Martin (1967) and is the simplest of the methods available. The method of partial totals, used by Wunsch (1966), and iterative techniques such as that used by Murphy and Nagnur (1972) or the one adopted in this work to estimate the transformed Gompertz parameters, can also be used. For the purposes for which estimates of the ordinary Gompertz parameters are required, however, the method of selected points is adequate.

The method is used to fit the Gompertz function to cumulative fertility. Three points or exact ages (x_o, x_1 and x_2) are used which must be equidistant:

$$x_1 - x_o = x_2 - x_1 = n$$

The first point, x_o, is taken as the origin. Taking natural logarithms of the Gompertz function (given in equation 2.1) gives

$$\ln F(x) = \ln F + B^{x-x_o} \ln A \qquad \text{A5.3.1}$$

and substituting x_o, x_1 and x_2 for x gives

$$\ln F(x_o) = \ln F + \ln A \qquad A5.3.2$$

$$\ln F(x_1) = \ln F + B^n \ln A \qquad A5.3.3$$

$$\ln F(x_2) = \ln F + B^{2n} \ln A \qquad A5.3.4$$

These three equations can be solved for F, A and B. Subtracting A5.3.2 from A5.3.3 gives

$$\ln F(x_1) - \ln F(x_o) = (B^n - 1) \ln A \qquad A5.3.5$$

Similarly, subtracting A5.3.3 from A5.3.4 gives

$$\ln F(x_2) - \ln F(x_1) = B^n(B^n - 1) \ln A \qquad A5.3.6$$

Dividing A5.3.6 by A5.3.5 results in

$$B^n = \frac{\ln F(x_2) - \ln F(x_1)}{\ln F(x_1) - \ln F(x_o)}$$

and rearranging A5.3.5 gives

$$\ln A = \frac{\ln F(x_1) - \ln F(x_o)}{B^n - 1}$$

$$= \frac{(\ln F(x_1) - \ln F(x_o))^2}{\ln F(x_2) - 2\ln F(x_1) + \ln F(x_o)} \quad \text{on substitution.}$$

From equation A5.3.2

$$\ln F = \ln F(x_o) - \ln A$$

$$= \frac{(\ln F(x_1))^2 - \ln F(x_o).\ln F(x_2)}{2\ln F(x_1) - \ln F(x_o) - \ln F(x_2)} \text{ on substitution.}$$

APPENDIX 5.4

TABLES OF CUMULATIVE FERTILITY RATES USED IN ANALYSES

The following data are used in the analyses in Chapter 5. They are presented here in their cumulative form.

Table A5.4.1: Cumulative fertility rates for simulated data

Exact age	Stage of fertility decline 1	2	3	4	5
15	.00713	.00649	.00216	.00257	.00123
20	.15724	.12881	.11169	.09259	.07806
25	.47364	.45344	.46572	.42296	.39586
30	.74325	.74006	.76046	.72598	.71582
35	.90154	.90309	.91333	.90461	.89725
40	.97007	.97299	.97413	.97078	.97161
45	.99599	.99497	.99698	.99533	.99506
50	.99964	.99983	.99957	.99977	.99969
55	.99999	1.00001	1.00000	1.00000	1.00000

Note that only data to age 50 are used in the analysis.

Table A5.4.2: Cumulative fertility rates by birth cohort of women, Sweden 1870/71 to 1915/16

Exact age	Cohort				
	1870/71	1875/76	1880/81	1885/86	1890/91
15	.0005	.0006	.0004	.0006	.0009
20	.1202	.1291	.1500	.1553	.1747
25	.7931	.8322	.8419	.8594	.8097
30	1.7947	1.8143	1.7617	1.6718	1.5543
35	2.7298	2.6962	2.5002	2.3396	2.1085
40	3.4171	3.2849	3.0001	2.7497	2.4178
45	3.6825	3.5058	3.1783	2.8812	2.5127
50	3.6994	3.5197	3.1878	2.8884	2.5178

Exact age	Cohort				
	1895/6	1900/01	1905/06	1910/11	1915/16
15	.0008	.0008	.0009	.0007	.0007
20	.1652	.1635	.1488	.1505	.1443
25	.7696	.7076	.5926	.5523	.6128
30	1.3903	1.2370	1.0524	1.0486	1.2969
35	1.8253	1.6186	1.4428	1.5625	1.7483
40	2.0655	1.8519	1.7450	1.8166	1.9492
45	2.1442	1.9322	1.8232	1.8720	1.9952
50	2.1492	1.9365	1.8262	1.8739	1.9970

Table A5.4.3: Cumulative fertility rates for native white women in the USA, cohorts 1899/1900 to 1904/05

Exact age	Cohort 1899/1900	1900/01	1901/02	1902/03	1903/04	1904/05
20	.244	.256	.280	.285	.285	.286
25	1.066	1.066	1.068	1.072	1.051	1.017
30	1.803	1.738	1.716	1.703	1.677	1.621
35	2.270	2.171	2.119	2.118	2.080	2.024
40	2.538	2.416	2.364	2.358	2.320	2.275
45	2.614	2.492	2.443	2.439	2.404	2.355

Table A5.4.4: Cumulative fertility rates by birth cohort of women, Canada 1911 to 1916

Exact age	Cohort 1911	1912	1913	1914	1915	1916
15	.003	.003	.003	.003	.003	.003
20	.247	.238	.227	.224	.216	.213
25	.871	.868	.880	.898	.904	.922
30	1.606	1.637	1.696	1.747	1.752	1.776
35	2.233	2.276	2.367	2.416	2.400	2.403
40	2.605	2.653	2.757	2.803	2.780	2.778
45	2.714	2.762	2.867	2.908	2.880	2.874
49*	2.720	2.767	2.873	2.913	2.885	2.879

* Data available to exact age 49 only.

Table A5.4.4: Cumulative fertility rates by birth cohort of women, Canada 1911 to 1916

Exact age	Cohort 1911	1912	1913	1914	1915	1916
15	.003	.003	.003	.003	.003	.003
20	.247	.238	.227	.224	.216	.213
25	.871	.868	.880	.898	.904	.922
30	1.606	1.637	1.696	1.747	1.752	1.776
35	2.233	2.276	2.367	2.416	2.400	2.403
40	2.605	2.653	2.757	2.803	2.780	2.778
45	2.714	2.762	2.867	2.908	2.880	2.874
49*	2.720	2.767	2.873	2.913	2.885	2.879

* Data available to exact age 49 only.

APPENDIX 5.5

TABLES OF PARAMETER ESTIMATES FOR THE TRANSFORMED GOMPERTZ MODEL

The following tables give complete sets of results for the analyses discussed in Chapter 5. This includes estimates of P and Q, not discussed in Chapter 5. The mean square error, S/n, where S is the weighted sum of squared deviations (see Chapter 5) and n is the number of datapoints included, provides a measure of goodness of fit of the model. A good fit in this sense, however, does not necessarily imply good prediction of F.

Table A5.5.1: Estimates of the parameters for simulated data

Points included	P	Estimates of parameters				S/n x 10^9
		Q	α	β	F	
Stage 1						
15 to 50	.47098	.26737	.28378	1.31914	.99679	24493
15 to 45	.47035	.26709	.28200	1.32016	.99760	26473
15 to 40	.47024	.26713	.28169	1.32001	.99775	30856
15 to 35	.45832	.28009	.24821	1.27265	1.01712	18285
15 to 30	.43938	.29432	.19555	1.22308	1.05201	11938
Stage 2						
15 to 50	.44848	.25053	.22077	1.38417	.99686	20973
15 to 45	.44936	.24973	.22322	1.38737	.99675	22526
15 to 40	.44875	.25052	.22154	1.38422	.99751	26043
15 to 35	.43976	.26015	.19660	1.34651	1.01233	19148
15 to 30	.41562	.28126	.13012	1.26846	1.06186	09774
Stage 3						
15 to 50	.45627	.22523	.24249	1.49064	.99698	11696
15 to 45	.45637	.22571	.24276	1.48850	.99725	12886
15 to 40	.46049	.21986	.25429	1.51476	.99050	07815
15 to 35	.45699	.22345	.24452	1.49857	.99614	06231
15 to 30	.44177	.23608	.20217	1.44357	1.02366	01149
Stage 4						
15 to 50	.41485	.23237	.12801	1.45942	.99552	14275
15 to 45	.41493	.23218	.12823	1.46025	.99533	13665
15 to 40	.41627	.23012	.13191	1.46917	.99291	15420
15 to 35	.40696	.24140	.10643	1.42130	1.01002	02548
15 to 30	.39959	.24784	.08631	1.39498	1.02535	01647
Stage 5						
15 to 50	.38966	.22878	.05924	1.47500	.99548	15305
15 to 45	.38947	.22820	.05873	1.47755	.99583	15182
15 to 40	.39064	.22666	.06191	1.48429	.99325	16894
15 to 35	.38491	.23406	.04632	1.45219	1.00474	12116
15 to 30	.36409	.25318	-.01031	1.37364	1.05248	00838

Table A5.5.2: Estimates of the parameters for Swedish data

Points included		Estimates of parameters				S/n $\times 10^9$
	P	Q	α	β	F	
1870/71 cohort						
15 to 50	.20436	.32670	-.46238	1.11871	3.70674	024102
15 to 45	.20421	.32807	-.46285	1.11452	3.71329	012769
15 to 40	.20391	.33045	-.46377	1.10730	3.72509	007158
15 to 35	.20616	.32518	-.45685	1.12338	3.68369	000720
15 to 30	.21025	.31936	-.44435	1.14145	3.61762	004614
1875/76 cohort						
15 to 50	.22557	.31375	-.39820	1.15916	3.52359	006541
15 to 45	.22549	.31466	-.39843	1.15626	3.52717	002985
15 to 40	.22535	.31547	-.39884	1.15368	3.53102	002322
15 to 35	.22442	.31738	-.40161	1.14765	3.54486	001052
15 to 30	.23142	.30856	-.38083	1.17583	3.44894	001434
1880/81 cohort						
15 to 50	.25533	.30880	-.31130	1.17507	3.19038	032429
15 to 45	.25536	.30914	-.31122	1.17395	3.19193	035434
15 to 40	.25545	.30847	-.31094	1.17612	3.18907	040408
15 to 35	.25917	.30106	-.30030	1.20045	3.14509	029468
15 to 30	.24163	.32171	-.35090	1.13412	3.34757	000303
1885/86 cohort						
15 to 50	.28324	.30021	-.23227	1.20326	2.89289	029047
15 to 45	.28303	.30131	-.23286	1.19961	2.89639	027777
15 to 40	.28227	.30363	-.23499	1.19196	2.90526	024104
15 to 35	.28393	.30040	-.23033	1.20264	2.88957	029846
15 to 30	.30389	.28017	-.17488	1.27235	2.72492	000540
1890/91 cohort						
15 to 50	.31317	.28612	-.14929	1.25134	2.52178	049010
15 to 45	.31372	.28475	-.14776	1.25615	2.51731	063719
15 to 40	.31171	.29016	-.15332	1.23733	2.53430	048071
15 to 35	.30611	.29902	-.16873	1.20724	2.57408	027079
15 to 30	.29211	.31400	-.20753	1.15836	2.68178	003708
1895/96 cohort						
15 to 50	.34606	.28055	-.05934	1.27102	2.14773	002419
15 to 45	.34671	.27912	-.05758	1.27612	2.14421	003777
15 to 40	.34708	.27864	-.05657	1.27784	2.14266	000245
15 to 35	.34803	.27721	-.05398	1.28298	2.13766	000240
15 to 30	.35167	.27346	-.04407	1.29659	2.11643	002006
1900/01 cohort						
15 to 50	.34872	.30628	-.05210	1.18324	1.88448	005968
15 to 45	.34897	.30579	-.05141	1.18484	1.88348	006911
15 to 40	.34975	.30401	-.04930	1.19069	1.87938	005704
15 to 35	.35452	.29806	-.03633	1.21046	1.86006	000691
15 to 30	.36115	.29183	-.01829	1.23159	1.83018	000715

Table A5.5.2 continued

Points included	Estimates of parameters P	 Q	 α	 β	 F	S/n x 10^9
			1905/06 cohort			
15 to 50	.30770	.33601	-.16434	1.09061	1.83757	138586
15 to 45	.30600	.34046	-.16904	1.07745	1.84759	122616
15 to 40	.30005	.35388	-.18549	1.03881	1.88209	055100
15 to 35	.31444	.33280	-.14580	1.10021	1.80791	016388
15 to 30	.34350	.30500	-.06632	1.18743	1.67456	001099
			1910/11 cohort			
15 to 50	.28342	.30670	-.23176	1.18188	1.89434	830491
15 to 45	.28156	.31324	-.23698	1.16080	1.90969	844763
15 to 40	.27250	.33997	-.26247	1.07891	1.98023	546883
15 to 35	.22992	.40966	-.38528	.89242	2.31204	005335
15 to 30	.25433	.38368	-.31418	.95795	2.10980	002991
			1915/16 cohort			
15 to 50	.30916	.24749	-.16033	1.39638	2.00564	556682
15 to 45	.30831	.24958	-.16267	1.38798	2.01088	618571
15 to 40	.30581	.25824	-.16957	1.35388	2.03164	650280
15 to 35	.28773	.29192	-.21971	1.23127	2.15018	497754
15 to 30	.19575	.39596	-.48914	.92643	3.03656	018659

Table A5.5.3: Estimates of the parameters for US data

Points included	Estimates of parameters P	Q	α	β	F	S/n x 10^9
			1899/1900 cohort			
20 to 45	.39218	.26804	.06610	1.31660	2.60754	083687
20 to 40	.39416	.26480	.07150	1.32877	2.59764	080051
20 to 35	.40357	.25321	.09716	1.37352	2.55157	004275
			1900/01 cohort			
20 to 45	.40948	.26946	.11331	1.31132	2.48206	154311
20 to 40	.41278	.26458	.12234	1.32963	2.46785	137230
20 to 35	.42384	.25161	.15268	1.37986	2.41962	048762
			1901/02 cohort			
20 to 45	.42033	.27620	.14305	1.28663	2.43109	177237
20 to 40	.42458	.27005	.15472	1.30917	2.41378	143071
20 to 35	.43846	.25394	.19298	1.37064	2.35660	010089
			1902/03 cohort			
20 to 45	.42097	.27970	.14481	1.27405	2.42810	165919
20 to 40	.42088	.27673	.14455	1.28470	2.42935	197451
20 to 35	.43642	.26080	.18736	1.34401	2.36475	058510
			1903/04 cohort			
20 to 45	.42020	.28239	.14268	1.26447	2.39205	151384
20 to 40	.42465	.27593	.15490	1.28760	2.37420	110402
20 to 35	.43655	.26248	.18772	1.33760	2.32659	021150
			1904/05 cohort			
20 to 45	.41421	.29263	.12627	1.22886	2.34810	118379
20 to 40	.41699	.28852	.13387	1.24298	2.33695	117396
20 to 35	.43049	.27299	.17098	1.29833	2.28236	024130

Table A5.5.4: Estimates of the parameters for Canadian data

Points included	P	Q	α	β	F	S/n x 10^9
			1911 cohort			
15 to 45	.31234	.32285	-.15156	1.13058	2.74907	393175
15 to 40	.30628	.33656	-.16826	1.08897	2.80025	217540
15 to 35	.28801	.36374	-.21894	1.01132	2.95389	025581
15 to 30	.26901	.38350	-.27231	.95842	3.14122	000918
			1912 cohort			
15 to 45	.30700	.31641	-.16629	1.15071	2.79519	400669
15 to 40	.30107	.32907	-.18093	1.11149	2.84312	249875
15 to 35	.28494	.25475	-.22752	1.03633	2.98878	066813
15 to 30	.25429	.38725	-.31428	.94868	3.31050	001181
			1913 cohort			
15 to 45	.29950	.30880	-.18700	1.17507	2.89908	404472
15 to 40	.29502	.32072	-.19944	1.13718	2.94468	264823
15 to 35	.28117	.34292	-.23807	1.07027	3.07450	086485
15 to 30	.24763	.37969	-.33349	.96841	3.44279	002790
			1914 cohort			
15 to 45	.30214	.30100	-.17971	1.20065	2.93725	368351
15 to 40	.29814	.31146	-.19078	1.16649	2.97726	257968
15 to 35	.28535	.33204	-.22637	1.10250	3.09464	132892
15 to 30	.24290	.37790	-.34721	.97314	3.57424	000115
			1915 cohort			
15 to 45	.30640	.29647	-.16794	1.21580	2.90554	246150
15 to 40	.30295	.30535	-.17748	1.18629	2.93891	161992
15 to 35	.29442	.31861	-.20110	1.14377	3.01232	112840
15 to 30	.25904	.35650	-.30067	1.03142	3.37544	002691
			1916 cohort			
15 to 45	.31285	.29214	-.15017	1.23052	2.89575	164078
15 to 40	.30996	.29912	-.15813	1.20693	2.92208	109221
15 to 35	.30625	.30494	-.16835	1.18765	2.95201	110835
15 to 30	.27376	.33878	-.25890	1.08241	3.25764	004111

Table A5.5.4: Estimates of the parameters for Canadian data

Points included	P	Q	α	β	F	S/n x 10^9
1911 cohort						
15 to 45	.31234	.32285	-.15156	1.13058	2.74907	393175
15 to 40	.30628	.33656	-.16826	1.08897	2.80025	217540
15 to 35	.28801	.36374	-.21894	1.01132	2.95389	025581
15 to 30	.26901	.38350	-.27231	.95842	3.14122	000918
1912 cohort						
15 to 45	.30700	.31641	-.16629	1.15071	2.79519	400669
15 to 40	.30107	.32907	-.18093	1.11149	2.84312	249875
15 to 35	.28494	.25475	-.22752	1.03633	2.98878	066813
15 to 30	.25429	.38725	-.31428	.94868	3.31050	001181
1913 cohort						
15 to 45	.29950	.30880	-.18700	1.17507	2.89908	404472
15 to 40	.29502	.32072	-.19944	1.13718	2.94468	264823
15 to 35	.28117	.34292	-.23807	1.07027	3.07450	086485
15 to 30	.24763	.37969	-.33349	.96841	3.44279	002790
1914 cohort						
15 to 45	.30214	.30100	-.17971	1.20065	2.93725	368351
15 to 40	.29814	.31146	-.19078	1.16649	2.97726	257968
15 to 35	.28535	.33204	-.22637	1.10250	3.09464	132892
15 to 30	.24290	.37790	-.34721	.97314	3.57424	000115
1915 cohort						
15 to 45	.30640	.29647	-.16794	1.21580	2.90554	246150
15 to 40	.30295	.30535	-.17748	1.18629	2.93891	161992
15 to 35	.29442	.31861	-.20110	1.14377	3.01232	112840
15 to 30	.25904	.35650	-.30067	1.03142	3.37544	002691
1916 cohort						
15 to 45	.31285	.29214	-.15017	1.23052	2.89575	164078
15 to 40	.30996	.29912	-.15813	1.20693	2.92208	109221
15 to 35	.30625	.30494	-.16835	1.18765	2.95201	110835
15 to 30	.27376	.33878	-.25890	1.08241	3.25764	004111

APPENDIX 6.1

ESTIMATES OF THE PARAMETERS OF THE TRANSFORMED GOMPERTZ MODEL

This appendix contains the full results of the fits of the transformed Gompertz model obtained under the two weighting systems described in Chapter 6. This includes the parameters P and Q, not presented in Chapter 6, and results for the cohort aged 45-49 which in the case of Bangladesh and West New Guinea is badly affected by omissions. The mean square errors, S/n, where S is the weighted sum of squared deviations (see Chapter 5) and n is the number of datapoints involved, is also presented. Whereas in Chapter 5 this provided a measure of goodness of fit of the model, it should be regarded here as more of a measure of the extent of reporting errors in the data. For the results obtained using the second set of weights, n is one less than for the results for the same cohort but using the first set of weights, because the final point is fixed.

Table A6.1.1: Estimates of the parameters; Bangladesh Fertility Survey, 1975

Cohort: age at survey	Estimates of parameters					S/n
	P	Q	α	β	F	
			a) equal weights			
30-34	.46088	.39299	.25539	.93396	7.62113	.00277
35-39	.41412	.40349	.12603	.90761	7.79200	.00949
40-44	.41810	.40736	.13693	.89806	7.53612	.00684
45-49	.38658	.41991	.05086	.86772	6.55879	.00186
			b) infinite weight to last point			
30-34	.50898	.32705	.39254	1.11766	6.99541	.01640
35-39	.41311	.42996	.12325	.84406	7.86493	.02026
40-44	.42502	.47944	.15594	.73514	7.66322	.07122
45-49	.39434	.47414	.07198	.74625	6.53242	.03774

Table A6.1.2: Estimates of the parameters; Sri Lanka Fertility Survey, 1975, 0-5 years education

Cohort: age at survey	Estimates of parameters					
	P	Q	α	β	F	S/n
	a) equal weights					
30-34	.41353	.36473	.12439	1.00861	5.23391	.00076
35-39	.39533	.37778	.07468	.97344	5.98055	.00426
40-44	.39848	.35178	.08328	1.04474	5.93519	.00374
45-49	.35357	.36227	-.03890	1.01536	6.07164	.00219
	b) infinite weight to last point					
30-34	.42186	.32731	.14725	1.11684	5.01931	.00657
35-39	.39399	.41512	.07104	.87919	6.10450	.01282
40-44	.40439	.35778	.09940	1.02784	5.87777	.00677
45-49	.36195	.39883	-.01612	.91922	6.05348	.01493

Table A6.1.3: Estimates of the parameters; Sri Lanka Fertility Survey, 1975, 6+ years education

Cohort: age at survey	Estimates of parameters					
	P	Q	α	β	F	S/n
	a) equal weights					
30-34	.24872	.38490	-.33033	.95477	4.27207	.00010
35-39	.27991	.28882	-.24159	1.24195	4.10442	.00031
40-44	.27064	.32428	-.26770	1.12616	4.18861	.00120
45-49	.27039	.32035	-.26842	1.13834	5.09426	.00218
	b) infinite weight to last point					
30-34	.25229	.39112	-.32004	.93875	4.28322	.00036
35-39	.29708	.34641	-.19372	1.06012	4.26523	.00837
40-44	.28135	.35396	-.23757	1.03856	4.19381	.00482
45-49	.27210	.31256	-.26357	1.16295	5.04123	.00335

Table A6.1.4: Estimates of the parameters; West New Guinea, 1961-62

Cohort: age at survey	Estimates of parameters					
	P	Q	α	β	F	S/n
a) equal weights						
30-34	.31433	.38089	-.14606	.96523	7.53636	.00078
35-39	.32721	.39999	-.11079	.91631	7.17161	.00063
40-44	.37272	.37988	.01316	.96790	6.47343	.00135
45-49*	.42093	.34716	.14469	1.05796	6.02543	.00061
b) infinite weight to last point						
30-34	.25709	.43926	-.30626	.82267	8.78789	.00792
35-39	.33606	.38273	-.08660	.96043	7.05181	.00248
40-44	.37798	.47906	.02745	.73593	6.75141	.06946
45-49*	.42380	.34809	.15256	1.05530	6.00464	.00101

* Point 15-19 not available; fit based on ages 20+.

APPENDIX 6.2

PARAMETER ESTIMATES FOR GRADUATED FERTILITY RATES

Results are presented here for the fits of the transformed Gompertz model to the graduated fertility rates derived in Chapter 6. The fitting procedure is identical to that used in Chapter 6 for reported rates. The weighting used is that with an infinite weight to the last point because the purpose of fitting to these data is to extrapolate beyond the last point to exact age 50. This is done for periods 0-4 and 5-9 years before the survey only, so that the maximum extent of extrapolation is 8 years.

Results for Bangladesh appear in Table A6.2.1. It is seen that the fit is not good and that there is a pattern of deviation with age. For both periods, rates for ages less than 25 are overestimated by the model whilst rates above age 25 are underestimated. This is due to the rates at younger ages being considerably lower than is expected from those at older ages, and could be the result of declining fertility at young ages or of use of an inappropriate pattern of fertility to redistribute reported mean parities for young cohorts. (The possibility of increasing fertility at older ages, which would produce the same pattern in the deviations, is rejected because of the clear fall in total fertility between the two periods.)

Table A6.2.1: Graduated and fitted cumulative fertility rates; Bangladesh Fertility Survey, 1975

	Period: years before survey					
	0-4			5-9		
Age	graduated (1)	fitted (2)	difference (1-2)	graduated (1)	fitted (2)	difference (1-2)
(10-14)*	(.04000)	(.05434)	(-.01434)	(.04000)	(.25213)	(-.21213)
15-19	.58500	.79405	-.20905	.81667	1.34389	-.52722
20-24	2.10661	2.21296	-.10635	2.47181	2.75236	-.28055
25-29	3.73741	3.66607	.07134	4.12564	4.05265	.07299
30-34	5.18386	5.00755	.17631	5.51834	5.25852	.25982
35-39	6.34459	6.20095	.14364	6.59954	6.40398	.19556
40-44	7.24053	7.15160	.08893	7.44850	7.44850	-
45-49	7.57921	7.57921	-		8.04484	
TF		7.64257			8.17738	
α		.03042			.12050	
β		.78572			.65948	
P		.37907			.41210	
Q		.45579			.51712	
S/n		.01991			.09355	
n		6			5	

* Age 10-14 not included in fitting procedure.

Results for Sri Lanka appear in Tables A6.2.2 and A6.2.3. Again the fits are poor compared to those obtained for cohort rates. For women with 0-5 years education the deviations show opposite trends with marked differences in the pattern parameters for the two periods. For women with 6+ years education, the pattern of deviations is the same for both periods and suggests a decline in fertility at younger ages.

Results for West New Guinea are shown in Table A6.2.4. These show opposite trends for the two periods, again accompanied by considerable differences in the pattern parameters.

Table A6.2.2: Graduated and fitted cumulative fertility rates; Sri Lanka Fertility Survey, 1975, 0-5 years education

	Period: years before survey					
	0-4			5-9		
Age	graduated (1)	fitted (2)	difference (1-2)	graduated (1)	fitted (2)	difference (1-2)
(10-14)*	(.00350)	(.00000)	(.00350)	(.00350)	(.01205)	(-.00855)
15-19	.05000	.01281	.03719	.13348	.35423	-.22075
20-24	.60315	.42698	.17617	1.04902	1.25431	-.20529
25-29	1.55587	1.52521	.03066	2.37766	2.31318	.06448
30-34	2.59988	2.75581	-.15593	3.51852	3.35556	.16296
35-39	3.53838	3.69475	-.15637	4.39299	4.30996	.08303
40-44	4.05397	4.18601	-.13204	5.07164	5.07164	-
45-49	4.29167	4.29167	-		5.40436	
TF		4.29630			5.45038	
α		-.46111			-.13217	
β		1.20436			.80954	
P		.20478			.31940	
Q		.29988			.44506	
S/n		.01660			.02570	
n		6			5	

* Age 10-14 not included in fitting procedure.

Table A6.2.3: Graduated and fitted cumulative fertility rates; Sri Lanka Fertility Survey, 1975, 6+ years education

	Period: years before survey					
	0-4			5-9		
Age	graduated (1)	fitted (2)	difference (-2)	graduated (1)	fitted (2)	difference (1-2)
(10-14)*	(.00000)	(.00095)	(-.00095)	(.00000)	(.00059)	(-.00059)
15-19	.06550	.11626	-.05076	.06550	.10904	-.04354
20-24	.52450	.63262	-.10812	.56652	.66152	-.09500
25-29	1.31801	1.38268	-.06467	1.48484	1.49887	-.01403
30-34	2.29376	2.17827	.11549	2.50369	2.39299	.11070
35-39	3.08275	2.91292	.16983	3.30635	3.20985	.09650
40-44	3.57610	3.47328	.10282	3.81718	3.81718	-
45-49	3.69280	3.69280	-		4.04485	
TF		3.71775			4.06895	
α		-.29863			-.31666	
β		.87506			.89876	
P		.25976			.25346	
Q		.41684			.40707	
S/n		.01200			.00654	
n		6			5	

* Age 10-14 not included in fitting procedure

Table A6.2.4: Graduated and fitted cumulative fertility rates; West New Guinea, 1961-62

	Period: years before survey					
	0-4			5-9		
Age	graduated (1)	fitted (2)	difference (1-2)	graduated (1)	fitted (2)	difference (1-2)
(10-14)*	(.00000)	(.00033)	(-.00033)	(.00100)	(.02347)	(-.02247)
15-19	.19800	.20806	-.01006	.19900	.58243	-.38343
20-24	1.55400	1.46386	.09014	1.55323	1.88581	-.33258
25-29	3.25611	3.24943	.00668	3.25475	3.29733	-.04258
30-34	4.83227	4.91444	-.08217	4.76022	4.60497	.15525
35-39	6.04927	6.20002	-.15075	5.80071	5.74122	.05949
40-44	6.92979	6.97317	-.04338	6.60146	6.60146	-
45-49	7.19098	7.19098	-		6.95550	
TF		7.20603			7.00074	
α		-.14112			-.01121	
β		1.04217			.83390	
P		.31614			.36375	
Q		.35269			.43435	
S/n		.00661			.05759	
n		6			5	

* Age 10-14 not included in fitting procedure.

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